Automatic Control (1)

By

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Lecture (2)

Syllabus

1 • Introduction.

2 • Fundamentals of mathematics.

3 • Transmission Circuits.

4 • Block Diagrams.

5 • Signal Flow Graphs.

6 • Analogue Computer.

7 • Mathematical Modeling of Physical Systems.

8 • State Space Analysis for Linear Systems.

9 • Time Domain Response.

10 • Transient Response.

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Syllabus

11 • Stability of Linear Control Systems.

12 • Discrete Time Systems.

13 • Z-Transform.

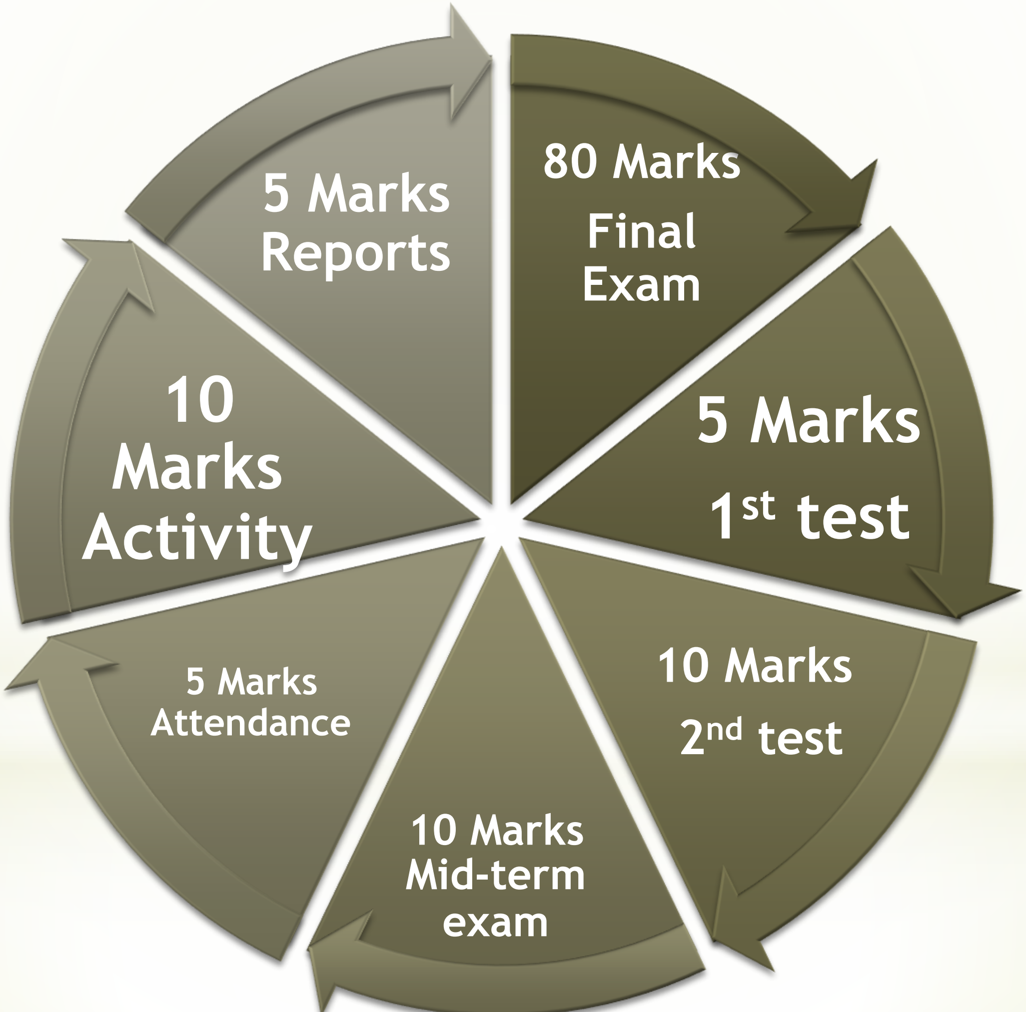
14 • Control System Design for Linear Dynamic Systems.

15 • Non-linear Control Systems.

16 • Stability of Non-linear Control Systems.

17 • Control System Design for Non-linear Dynamic Systems.

Marks Distribution Chart



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Block Diagram Fundamentals
&
Reduction Techniques

Introduction

- Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

OR

- A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.

Introduction

- The simplest form of the block diagram is the single ***block*, *with one input and one output***.

- The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the output.

- The arrows represent the direction of information or signal flow.

$$x \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow y$$

Introduction

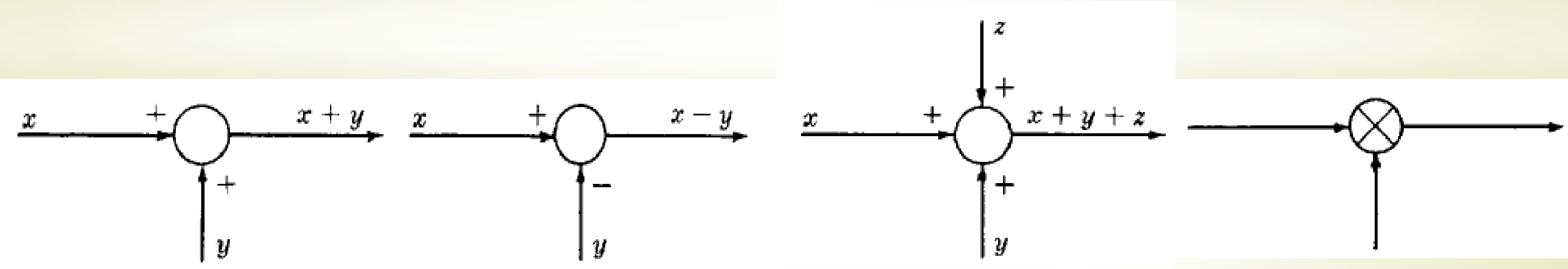
- The operations of addition and subtraction have a special representation.

- The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.

- Any number of inputs may enter a summing point.

- The output is the algebraic sum of the inputs.

- Some books put a cross in the circle.



Components of a Block Diagram for a Linear Time Invariant System

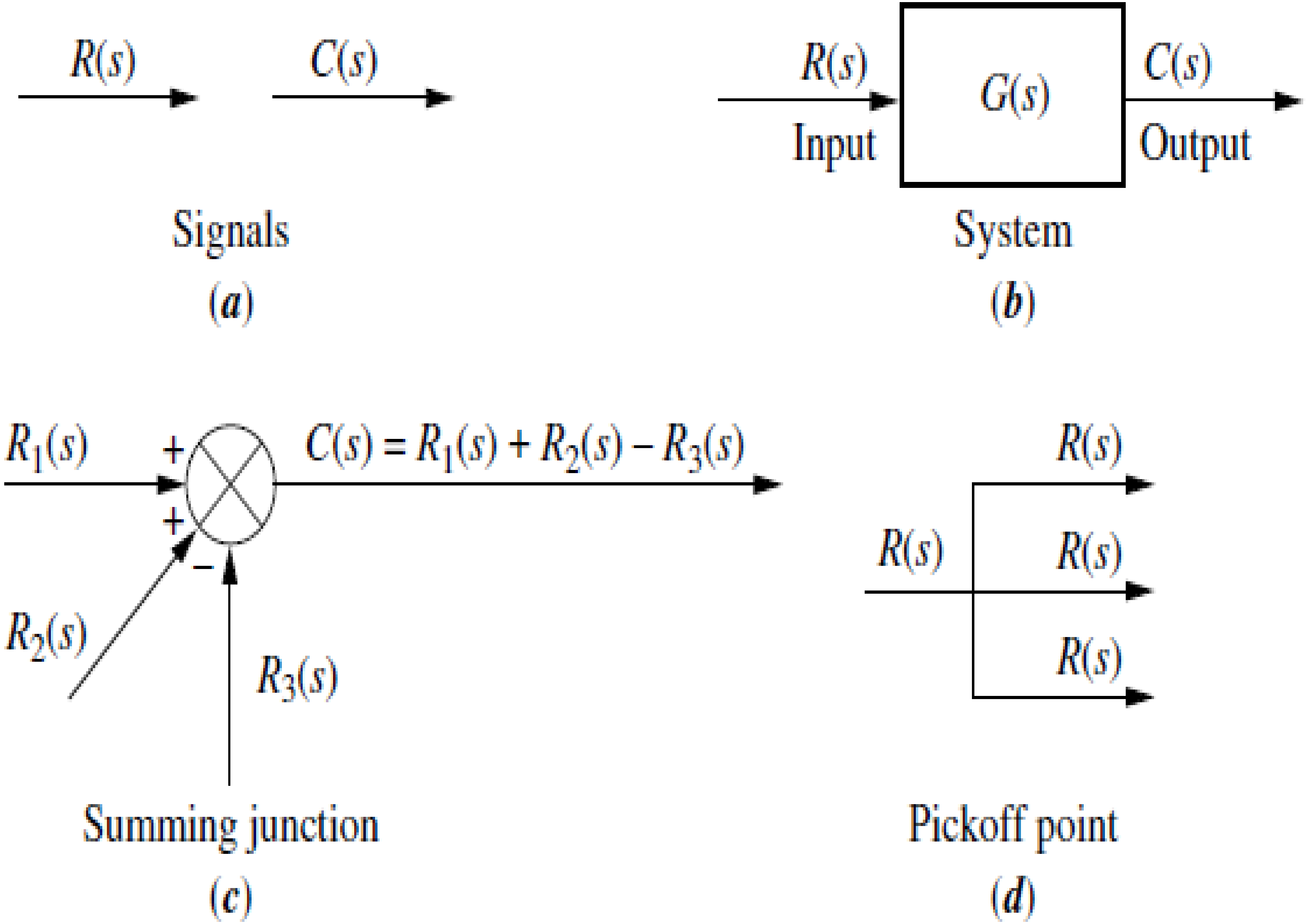
- System components are alternatively called elements of the system.
- Block diagram has four components:

1- Signals

2- System / block

3- Summing junction

4- Pick-off / Take-off point

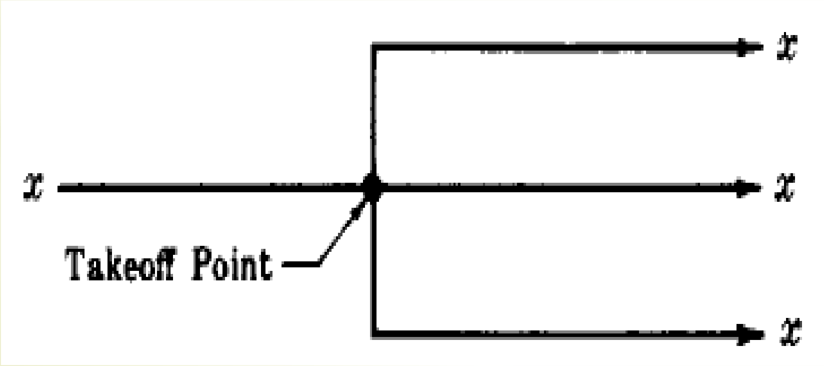


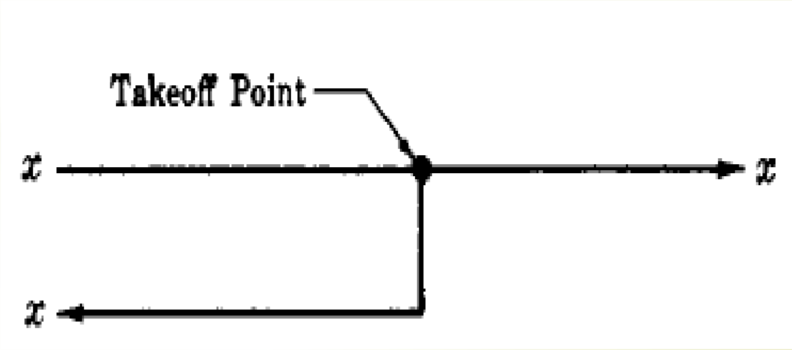
R(s)
C(s)
Signals
(a)
R(s)
Input
G(s)
C(s)
Output
System
(b)
R1(s)
+
+
−
C(s) = R1(s) + R2(s) − R3(s)
R2(s)
R3(s)
Summing junction
(c)
R(s)
R(s)
R(s)
R(s)
Pickoff point
(d)

- To have the same signal or variable be an input to more than one block or summing point, a take off point is used.

- Distributes the input signal, undiminished, to several output points.

- This permits the signal to proceed unaltered along several different paths to several destinations.

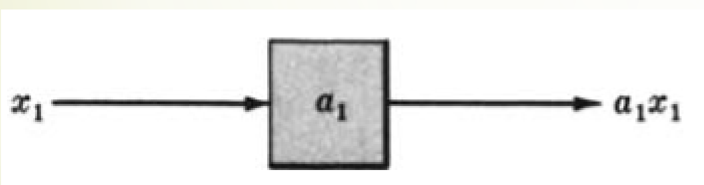


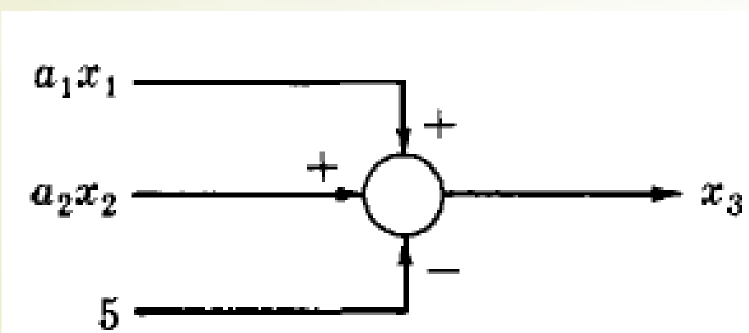


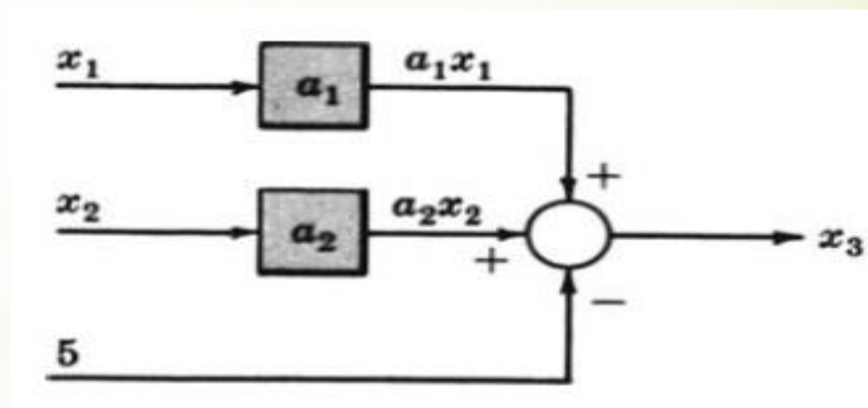
Example (1)

- Consider the following equations in which x_1, x_2, x_3, are variables, and a_1, a_2 *are* general coefficients or mathematical operators.

$$x_3 = a_1 x_1 + a_2 x_2 - 5$$



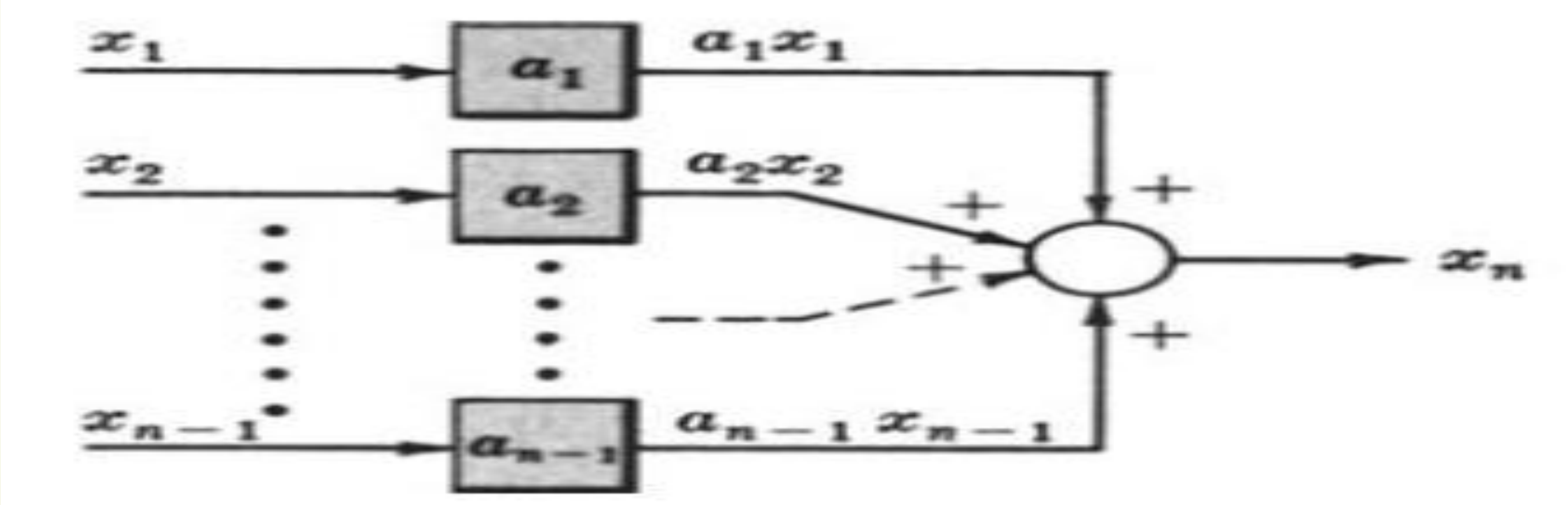




Example (2)

- Consider the following equations in which x_1, x_2,. . . , x_n, are variables, and a_1, a_2,. . . , a_n , *are* general coefficients or mathematical operators.

$$x_n = a_1 x_1 + a_2 x_2 + a_{n-1} x_{n-1}$$



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Example (3)

\- Draw the Block Diagrams of the following equations.

(1) $$x_2 = a_1 \frac{dx_1}{dt} + \frac{1}{b}\int x_1 dt$$

(2) $$x_3 = a_1 \frac{d^2 x_2}{dt^2} + 3\frac{dx_1}{dt} - bx_1$$

Topologies

- We will now examine some common topologies for interconnecting subsystems and derive the single transfer function representation for each of them.

- These common topologies will form the basis for reducing more complicated systems to a single block.

Cascade

- Any finite number of blocks in series may be algebraically combined by multiplication of transfer functions.
- That is, *n components or blocks with transfer functions* G_1 , G_2, . . . , G_n, *connected in cascade* are equivalent to a single element G with a transfer function given by

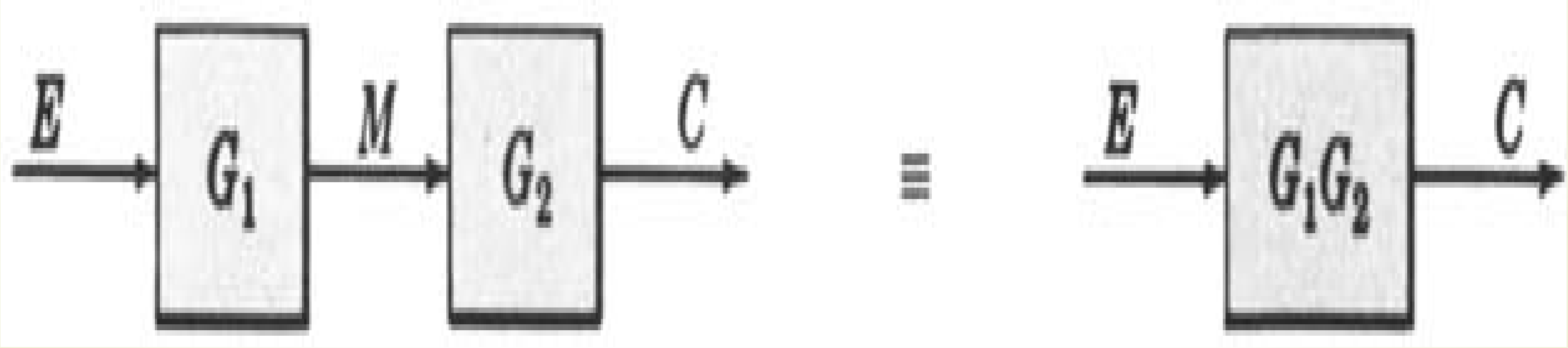
$$G = G_1 \cdot G_2 \cdot G_3 \cdots G_n = \prod_{i=1}^{n} G_i$$

Example

- Multiplication of transfer functions is *commutative; that is,*

$$G_iG_j = G_jG_i$$

for any i or *j* .



Cascade

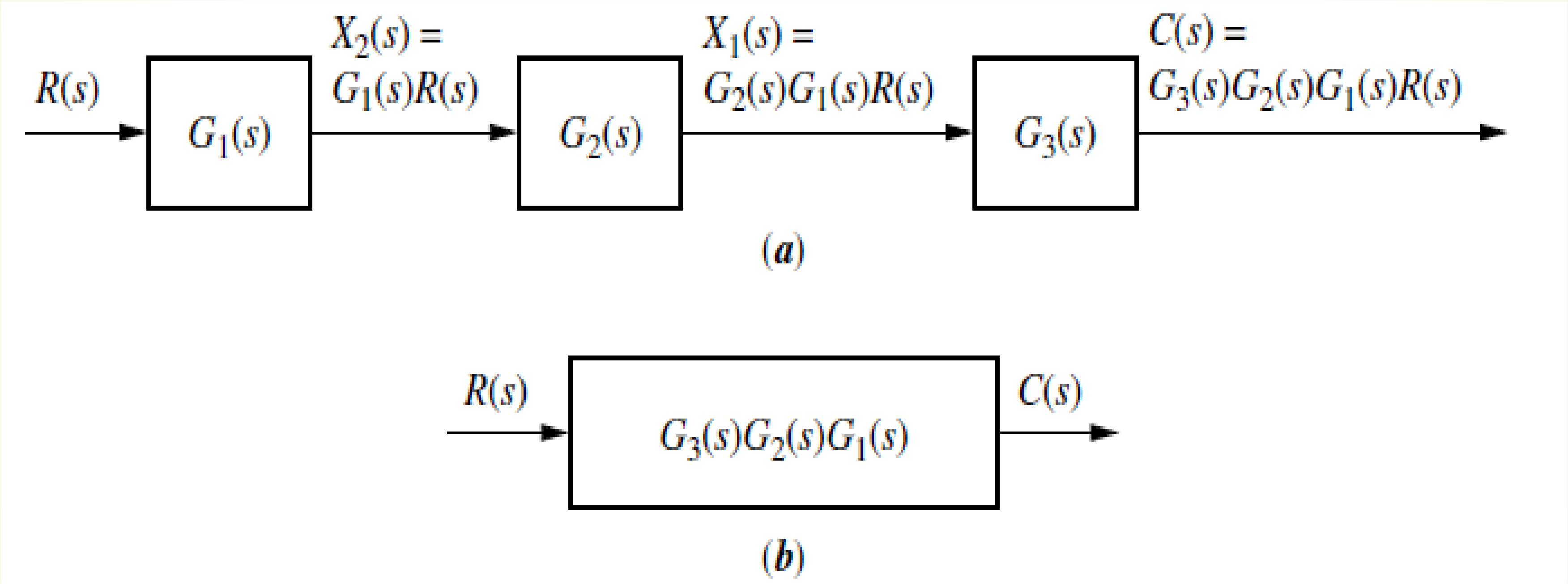


Figure:
a) Cascaded Subsystems.
b) Equivalent Transfer Function.

The equivalent transfer function is

$$G_e(s) = G_3(s)G_2(s)G_1(s)$$

Parallel Form

- Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.

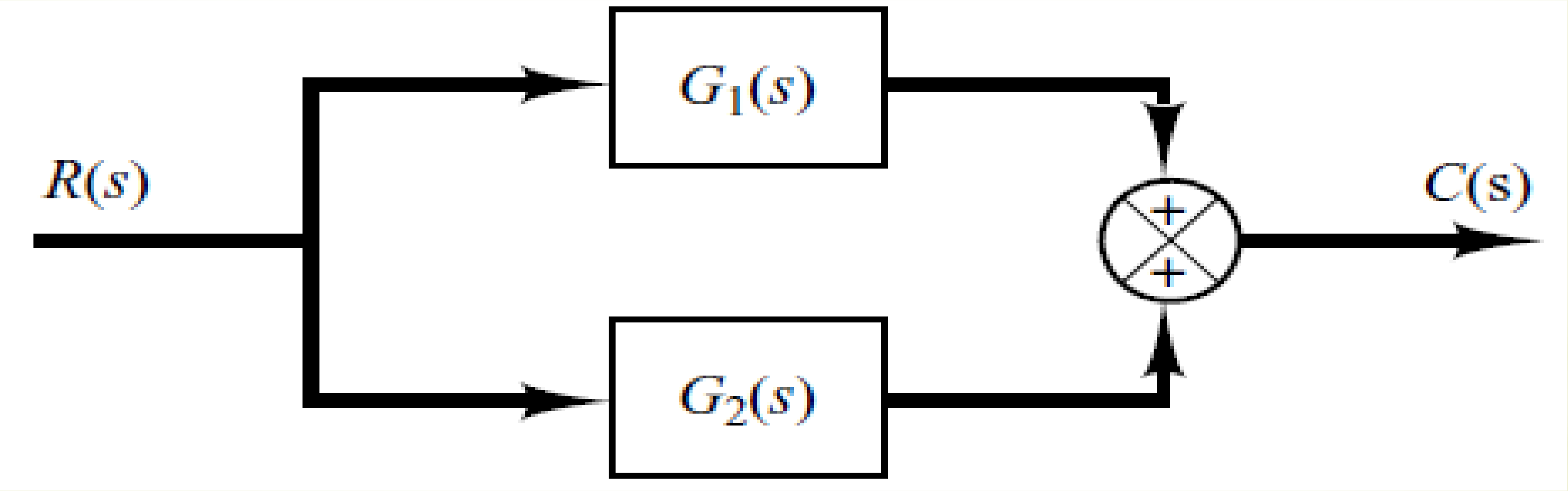


Figure: Parallel Subsystems.

Parallel Form

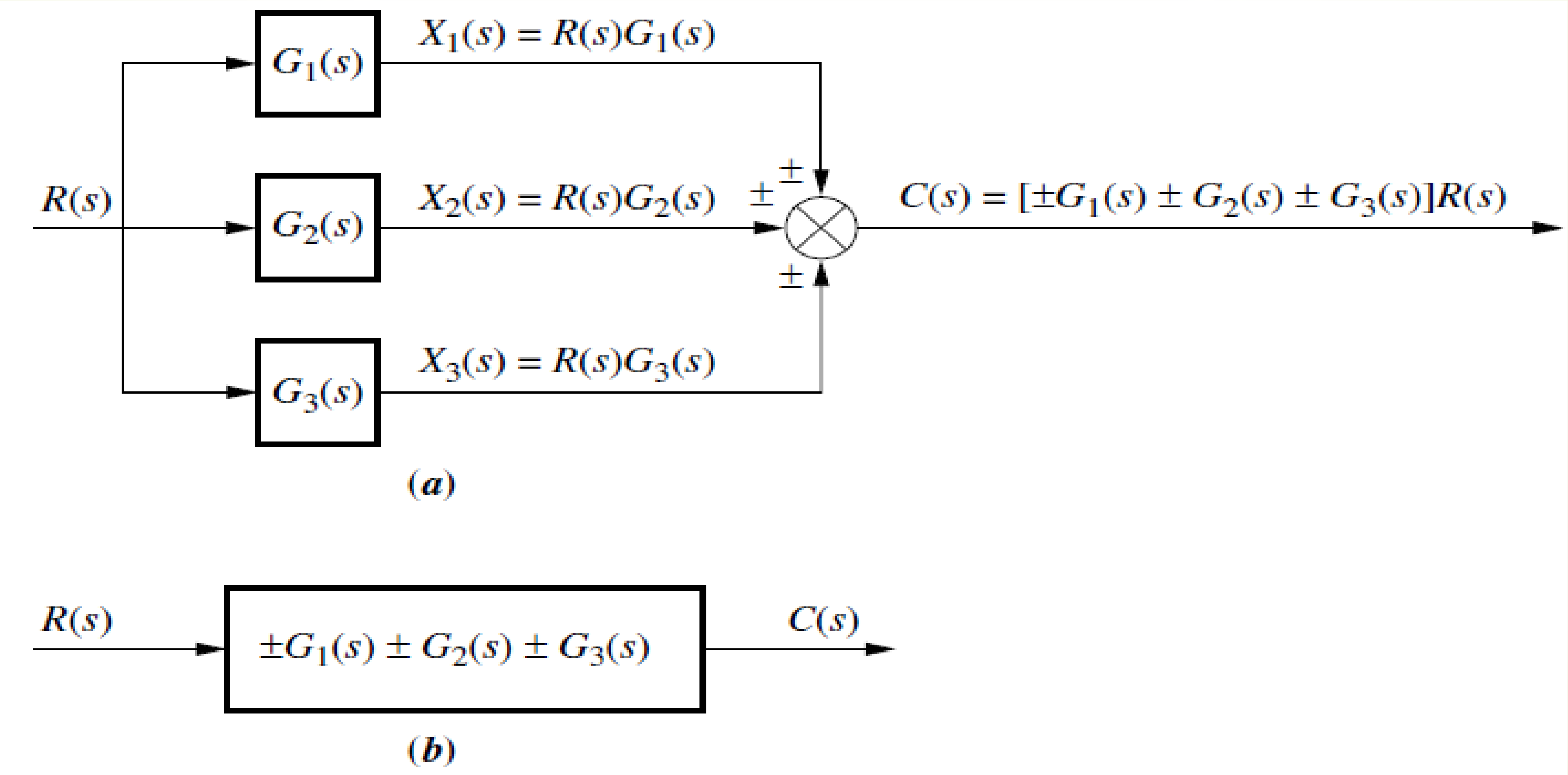


Figure:

a) Parallel Subsystems. b) Equivalent Transfer Function.

The equivalent transfer function is $\boxed{G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)}$

Feedback Form

- The third topology is the feedback form. Let us derive the transfer function that represents the system from its input to its output. The typical feedback system, shown in figure:

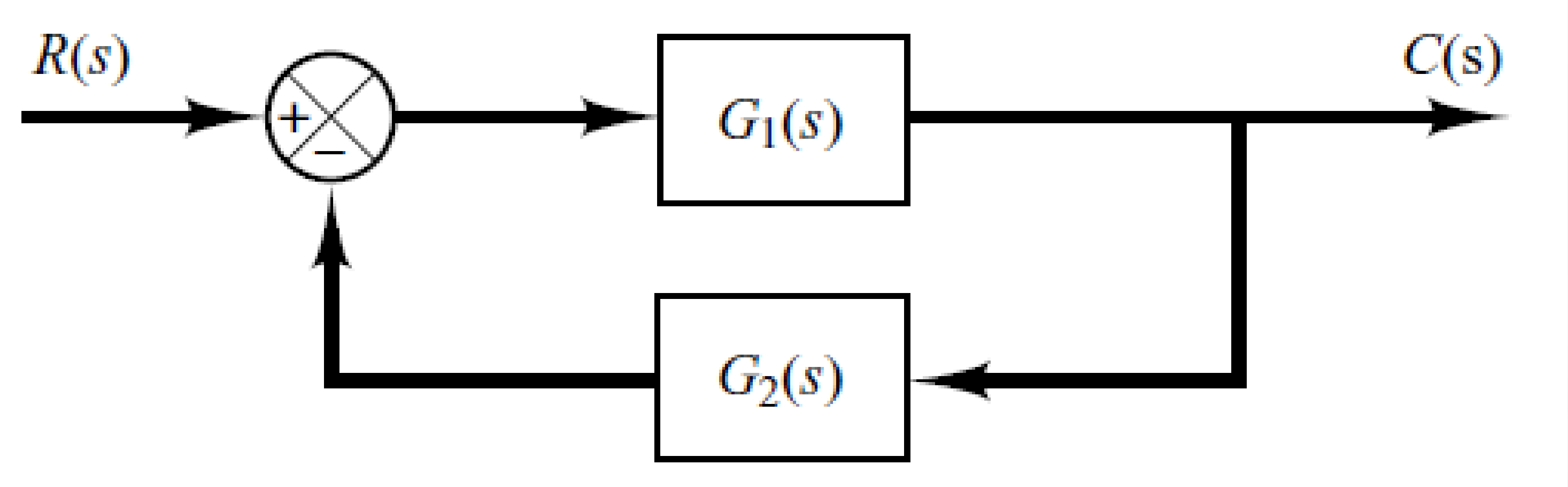


Figure: Feedback (Closed Loop) Control System.

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

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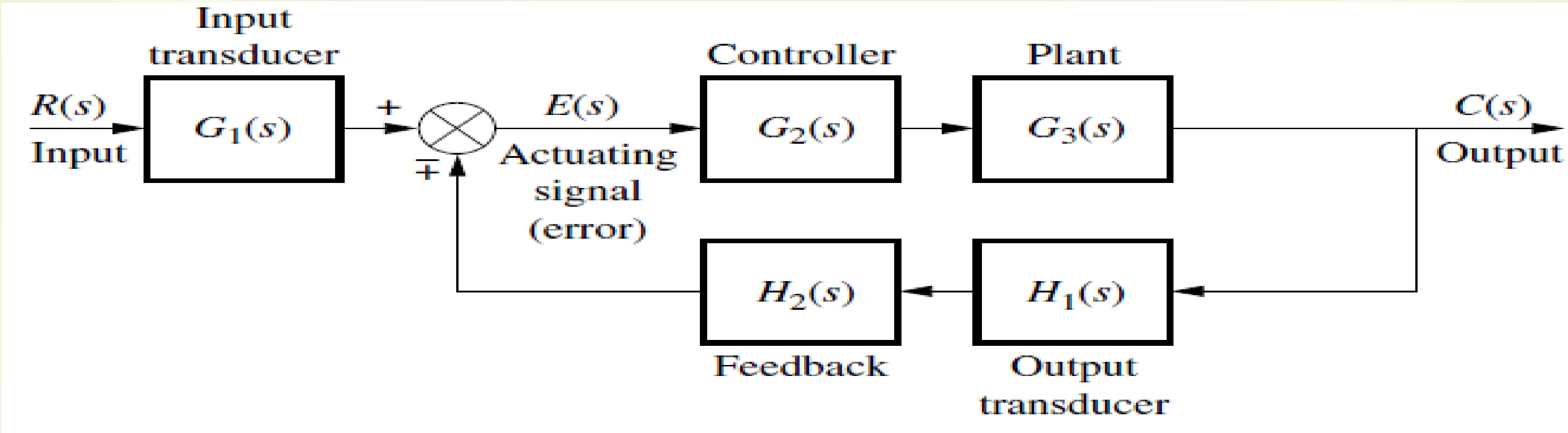
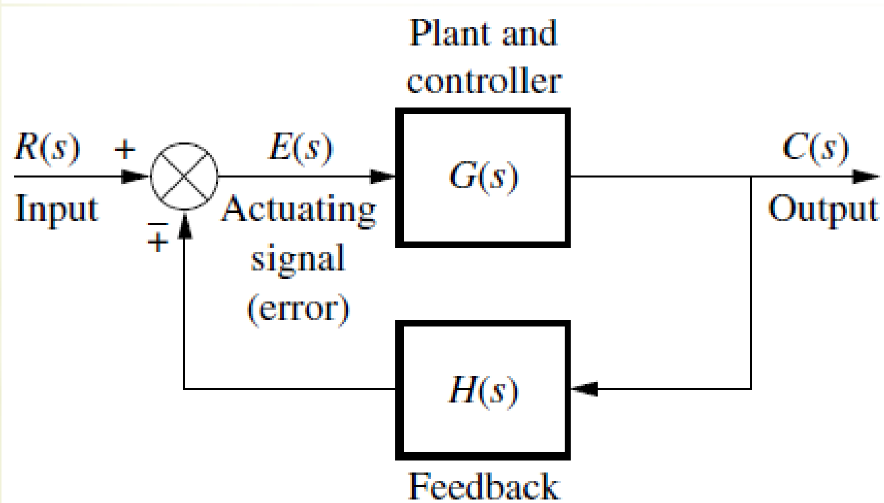
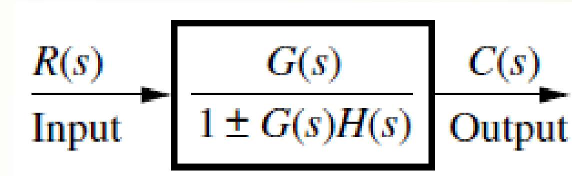
Feedback Form



(a)



(b)



(c)

Figure:

a) Feedback Control System.

b) Simplified Model or Canonical Form.

c) Equivalent Transfer Function.

The equivalent or closed-loop transfer function is

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Characteristic Equation

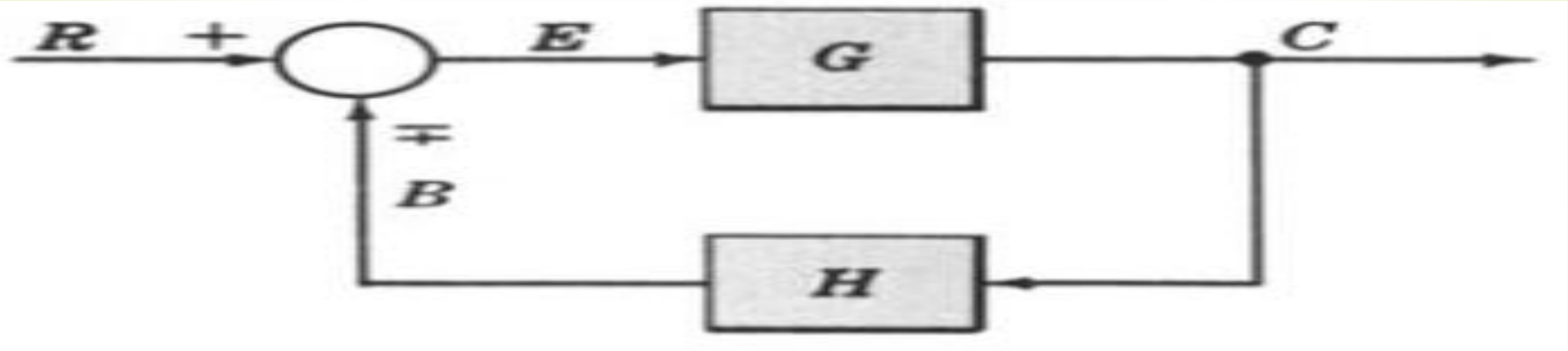
- The control ratio is the closed loop transfer function of the system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

- The denominator of closed loop transfer function determines the characteristic equation of the system.

- Which is usually determined as: $1 \pm G(s)H(s) = 0$

Canonical Form of a Feedback Control System



$G \equiv$ direct transfer function $\equiv$ forward transfer function

$H \equiv$ feedback transfer function

$GH \equiv$ loop transfer function $\equiv$ open-loop transfer function

$C/R \equiv$ closed-loop transfer function $\equiv$ control ratio $\quad \dfrac{C}{R} = \dfrac{G}{1 \pm GH}$

$E/R \equiv$ actuating signal ratio $\equiv$ error ratio $\quad \dfrac{E}{R} = \dfrac{1}{1 \pm GH}$

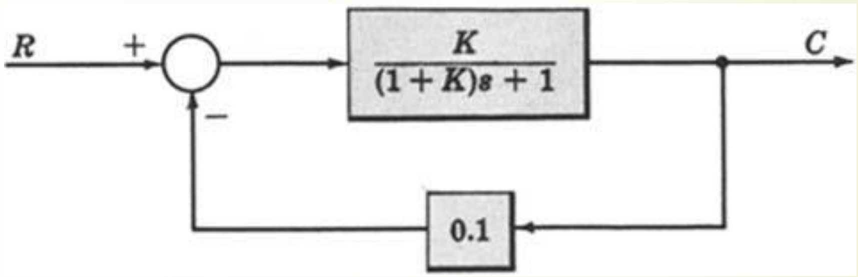
$B/R \equiv$ primary feedback ratio $\quad \dfrac{B}{R} = \dfrac{GH}{1 \pm GH}$

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

Example (4)

1. Open loop transfer function $\dfrac{B(s)}{E(s)} = G(s)H(s)$
2. Feed Forward Transfer function $\dfrac{C(s)}{E(s)} = G(s)$
3. Control ratio $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$
4. Feedback ratio $\dfrac{B(s)}{R(s)} = \dfrac{G(s)H(s)}{1+G(s)H(s)}$
5. Error ratio $\dfrac{E(s)}{R(s)} = \dfrac{1}{1+G(s)H(s)}$
6. Closed loop transfer function $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$
7. Characteristic equation $1+G(s)H(s) = 0$
8. Closed loop poles and zeros if K=10

$G(s)$



$H(s)$

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Characteristic Equation

$C/R \equiv$ closed-loop transfer function $\equiv$ control ratio

$$\frac{C}{R} = \frac{G}{1 \pm GH}$$

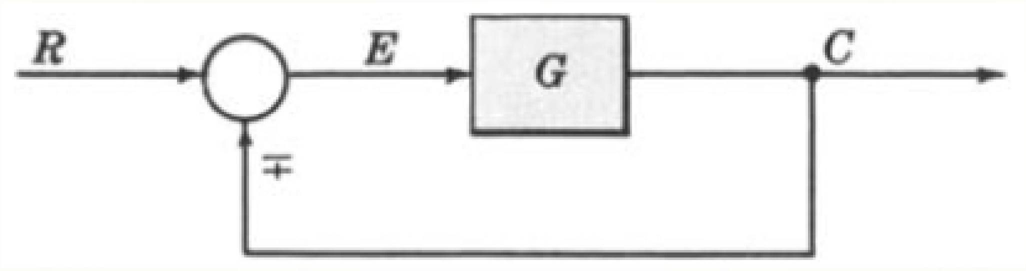
The denominator of C/R determines the *characteristic equation* of the system, which is usually determined from $1 \pm GH = 0$ or, equivalently,

$$D_{GH} \pm N_{GH} = 0$$

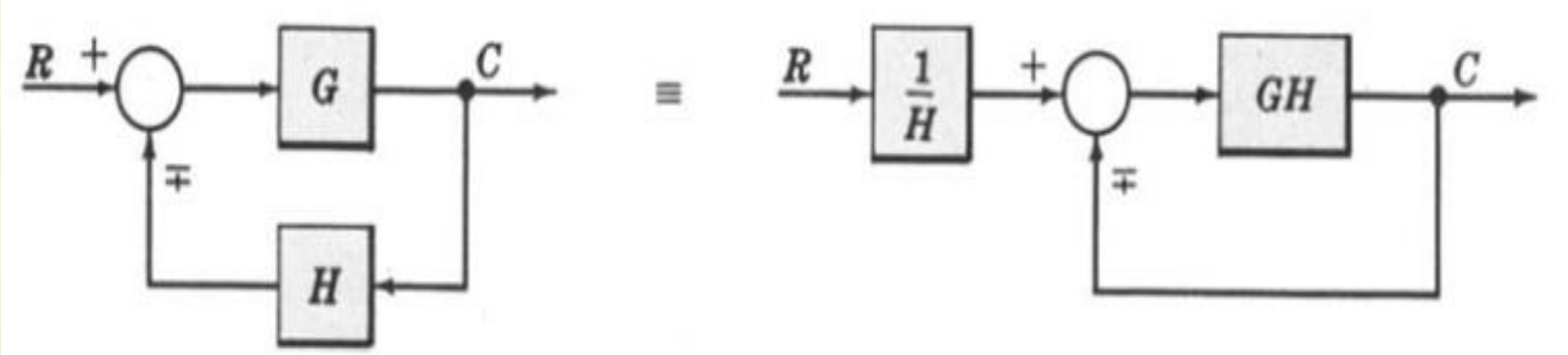
where D_{GH} is the denominator and N_{GH} is the numerator of GH

Unity Feedback System

A **unity feedback system** is one in which the primary feedback b is identically equal to the controlled output $H = 1$ for a linear, unity feedback system

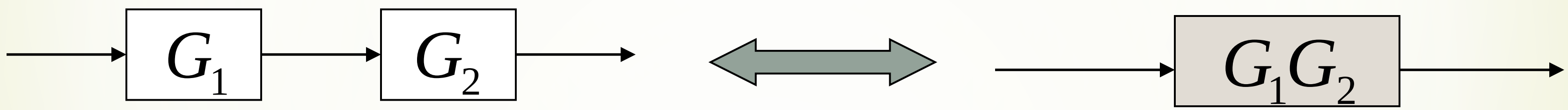


Any feedback system with only linear time-invariant elements can be put into the form of a unity feedback system by using Transformation 5.

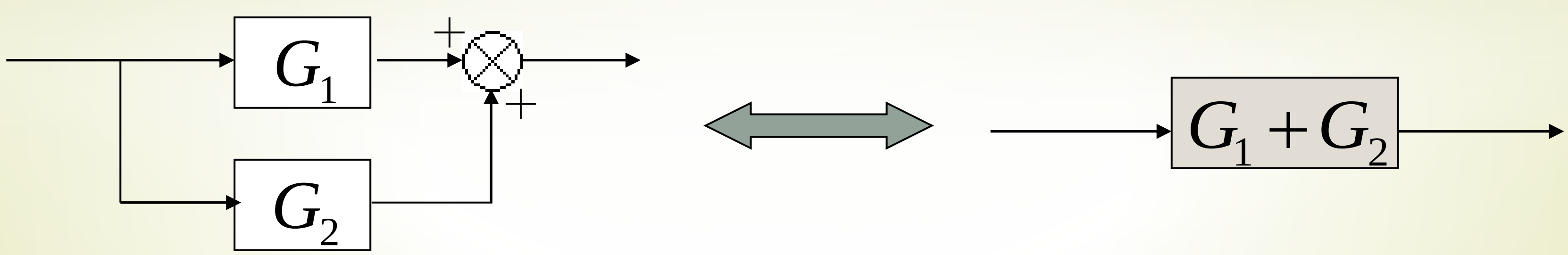


Reduction Techniques

1. Combining blocks in cascade

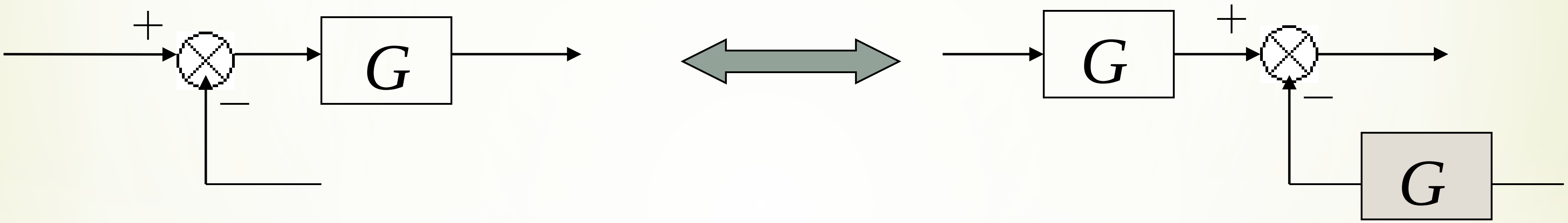


2. Combining blocks in parallel



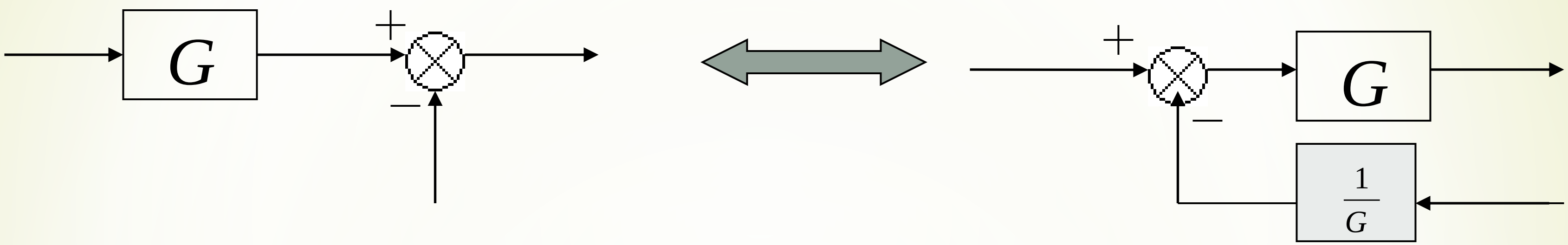
Reduction Techniques

3. Moving a summing point behind a block

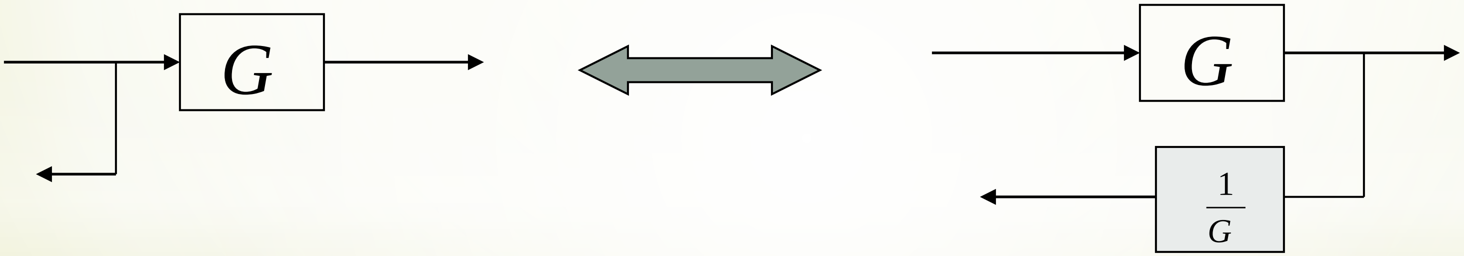


Reduction Techniques

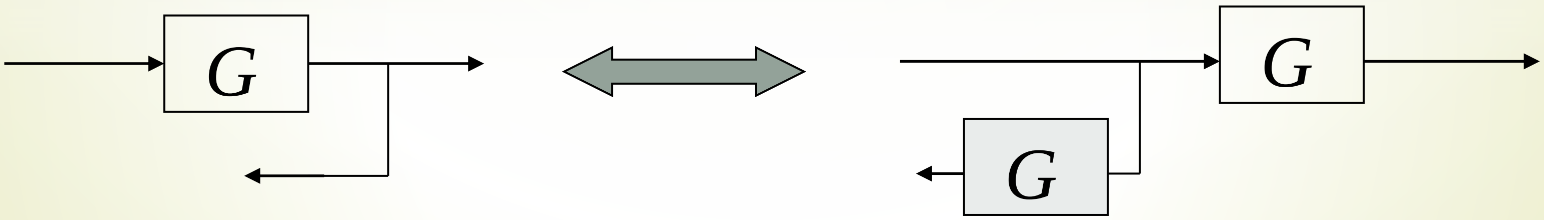
3. Moving a summing point ahead of a block



4. Moving a pickoff point behind a block

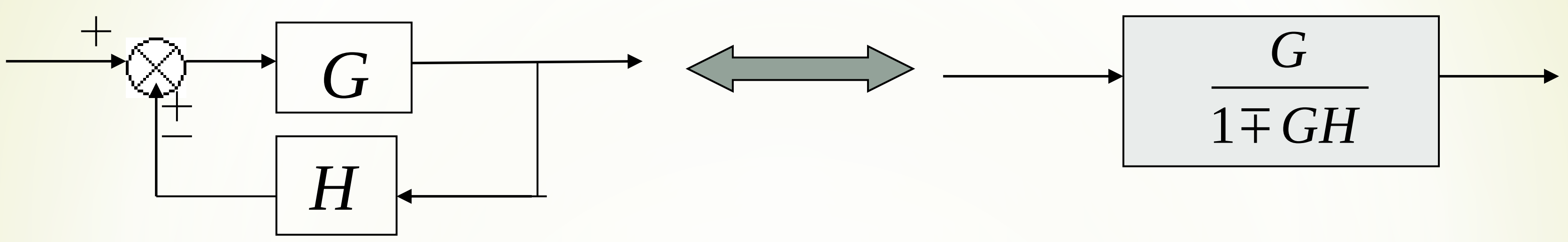


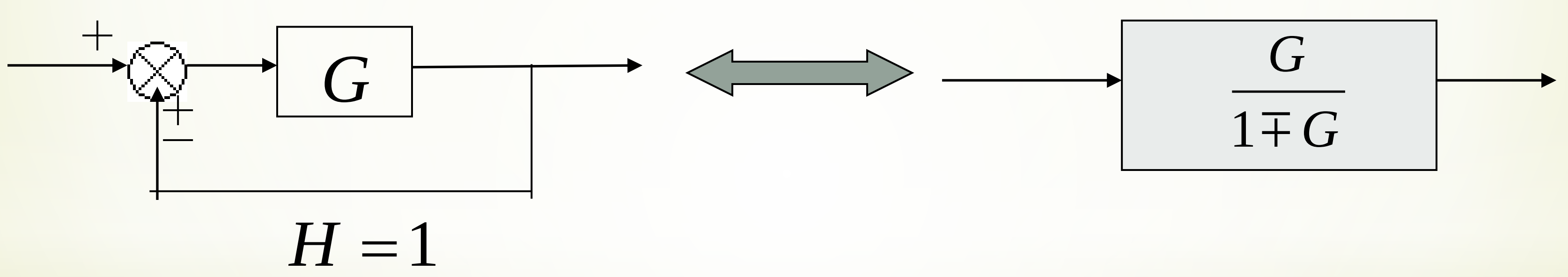
5. Moving a pickoff point ahead of a block



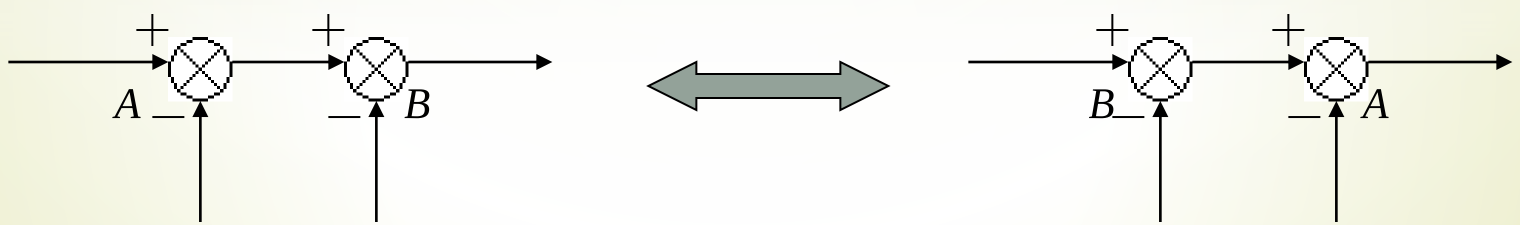
Reduction Techniques

6. Eliminating a feedback loop

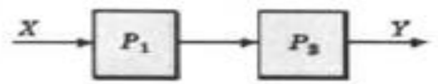
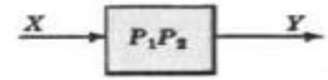
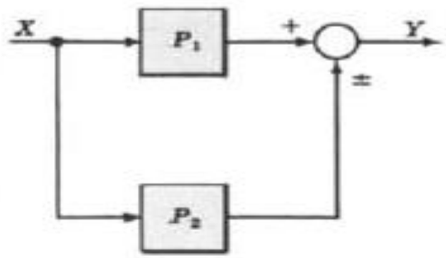
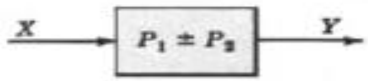
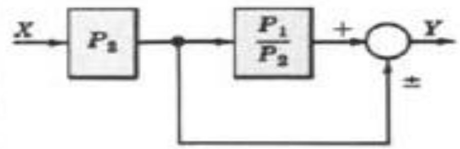
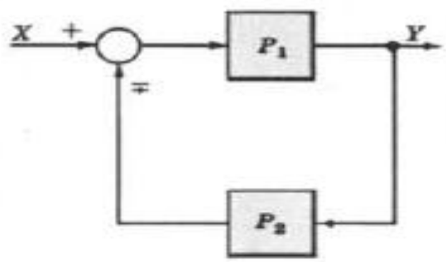
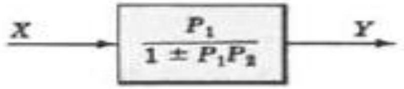
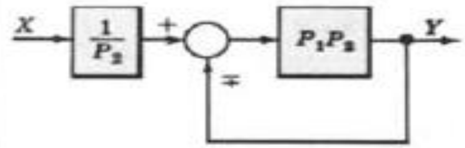




7. Swap with two neighboring summing points

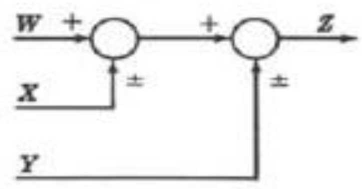
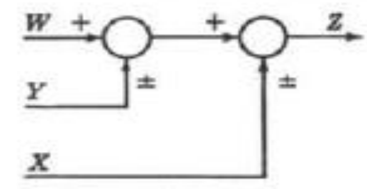
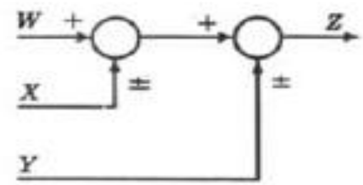
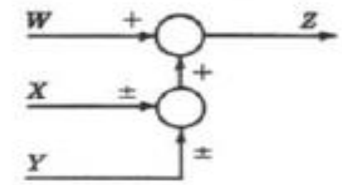
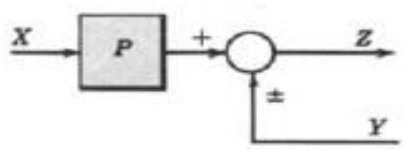
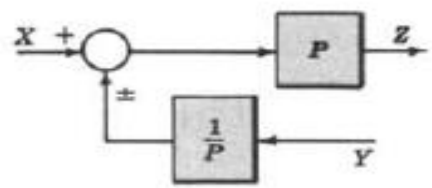
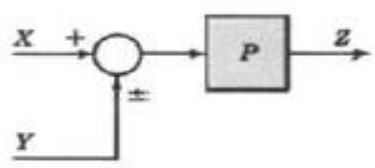
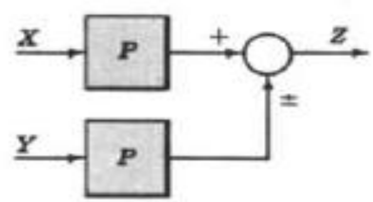


Block Diagram Transformation Theorems

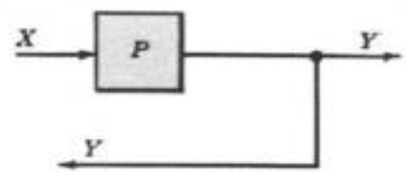
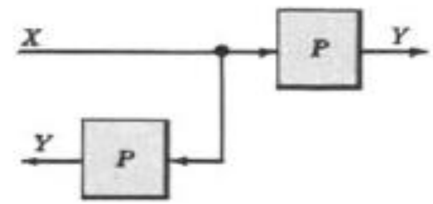
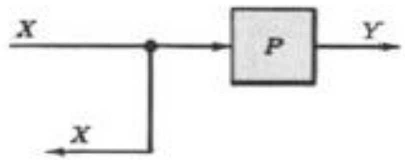
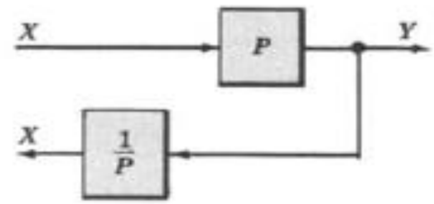
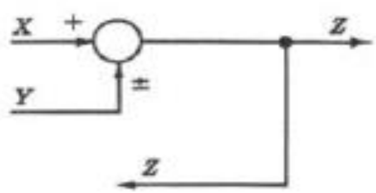
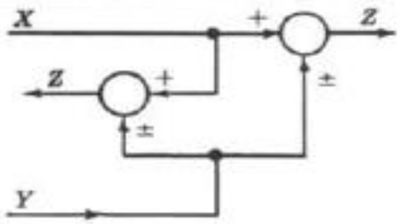
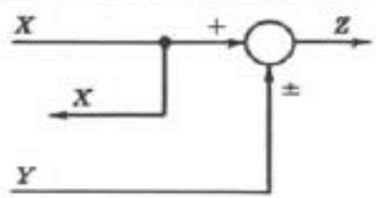
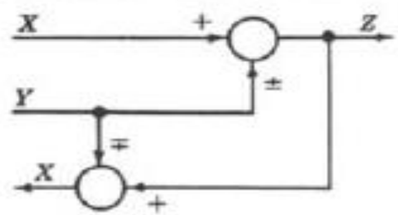
	Transformation	Equation	Block Diagram	Equivalent Block Diagram
1	Combining Blocks in Cascade	$Y = (P_1P_2)X$		
2	Combining Blocks in Parallel; or Eliminating a Forward Loop	$Y = P_1X \pm P_2X$		
3	Removing a Block from a Forward Path	$Y = P_1X \pm P_2X$		
4	Eliminating a Feedback Loop	$Y = P_1(X \mp P_2Y)$		
5	Removing a Block from a Feedback Loop	$Y = P_1(X \mp P_2Y)$		

The letter ***P* is used to represent any transfer function, and W, X , Y, Z denote any transformed** signals.

Block Diagram Transformation Theorems

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
6a	Rearranging Summing Points	$Z = W \pm X \pm Y$		
6b	Rearranging Summing Points	$Z = W \pm X \pm Y$		
7	Moving a Summing Point Ahead of a Block	$Z = PX \pm Y$		
8	Moving a Summing Point Beyond a Block	$Z = P[X \pm Y]$		

Block Diagram Transformation Theorems

	Transformation	Equation	Block Diagram	Equivalent Block Diagram
9	Moving a Takeoff Point Ahead of a Block	$Y = PX$		
10	Moving a Takeoff Point Beyond a Block	$Y = PX$		
11	Moving a Takeoff Point Ahead of a Summing Point	$Z = X \pm Y$		
12	Moving a Takeoff Point Beyond a Summing Point	$Z = X \pm Y$		

Reduction of Complicated Block Diagrams

The block diagram of a practical feedback control system is often quite complicated. It may include several feedback or feedforward loops, and multiple inputs. By means of systematic block diagram reduction, every multiple loop linear feedback system may be reduced to canonical form.

The following general steps may be used as a basic approach in the reduction of complicated block diagrams.

Step 1: Combine all cascade blocks using Transformation 1.

Step 2: Combine all parallel blocks using Transformation 2.

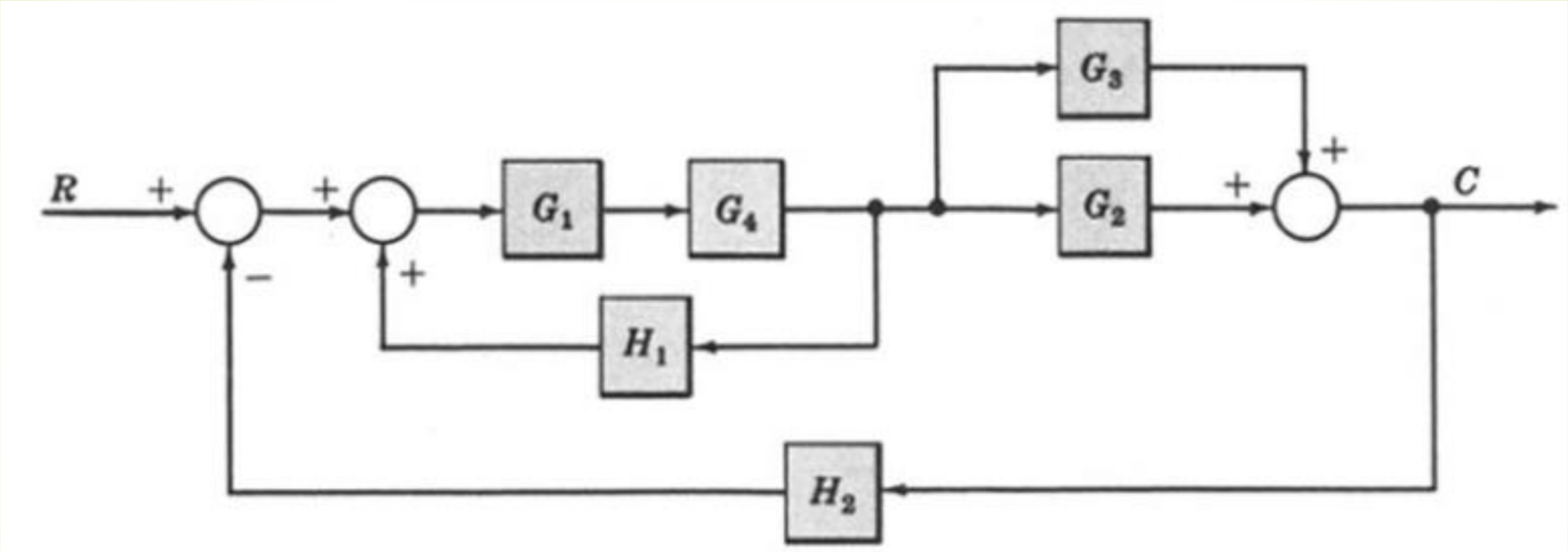
Step 3: Eliminate all minor feedback loops using Transformation 4.

Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12.

Step 5: Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.

Step 6: Repeat Steps 1 to 5 for each input, as required.

Example-5: Reduce the Block Diagram to Canonical



Step 1: Combine all cascade blocks using Transformation 1.

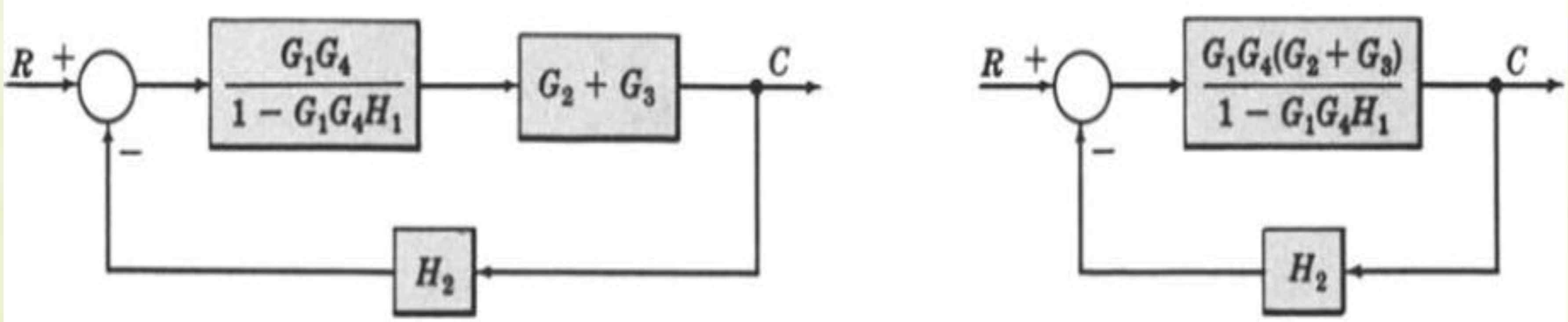
Step 2: Combine all parallel blocks using Transformation 2.

Example-5: Continue

Step 3: Eliminate all minor feedback loops using Transformation 4.

Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12. However in this example step-4 does not apply.

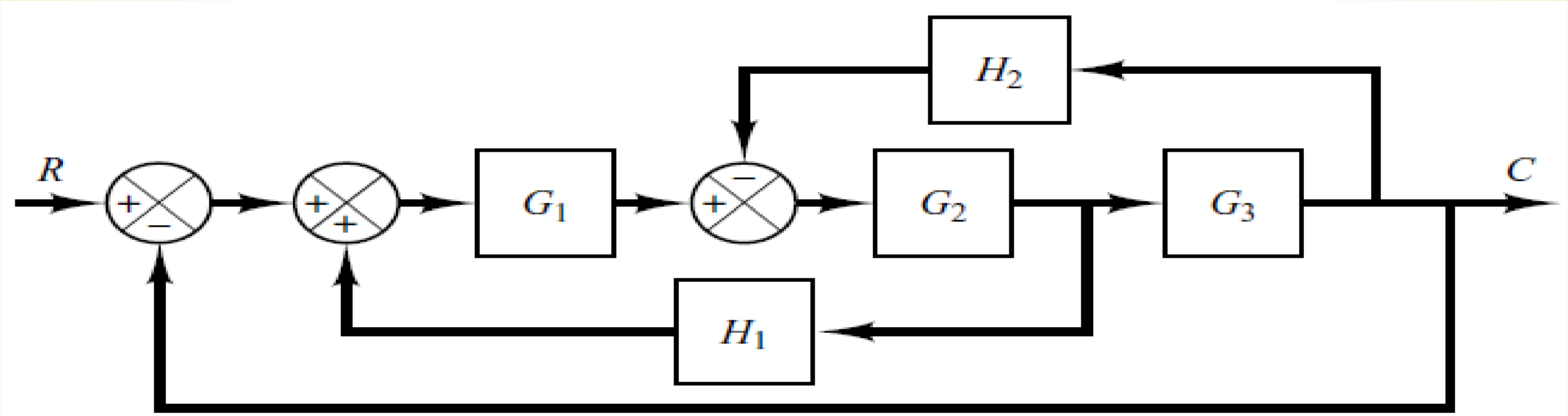
Step 5: Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.



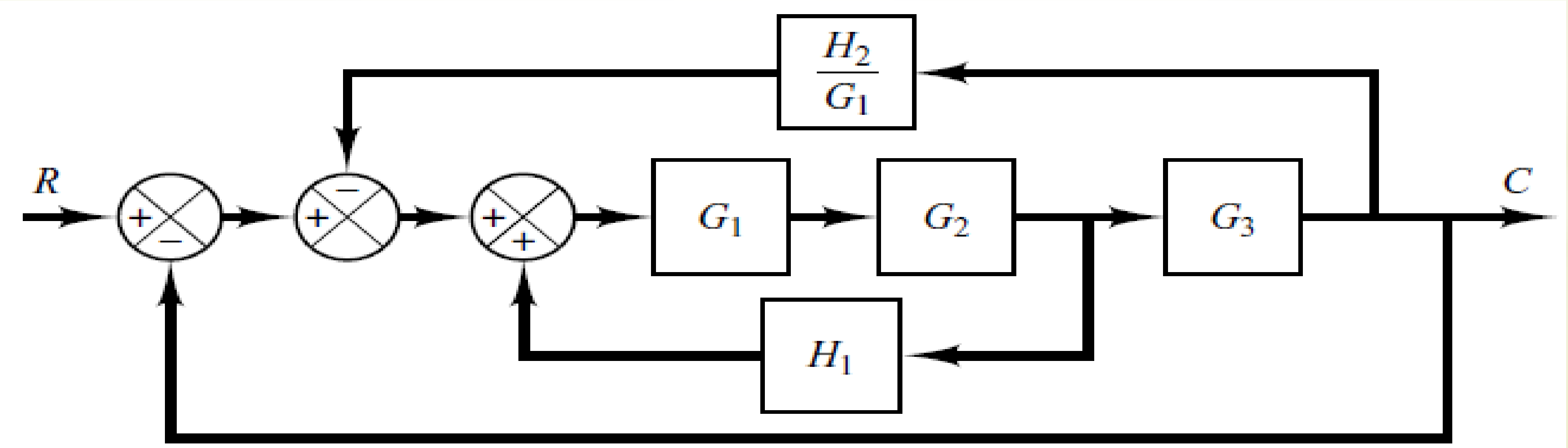
Step 6: Repeat Steps 1 to 5 for each input, as required.

However in this example step-6 does not apply.

Example-6: Simplify the Block Diagram.

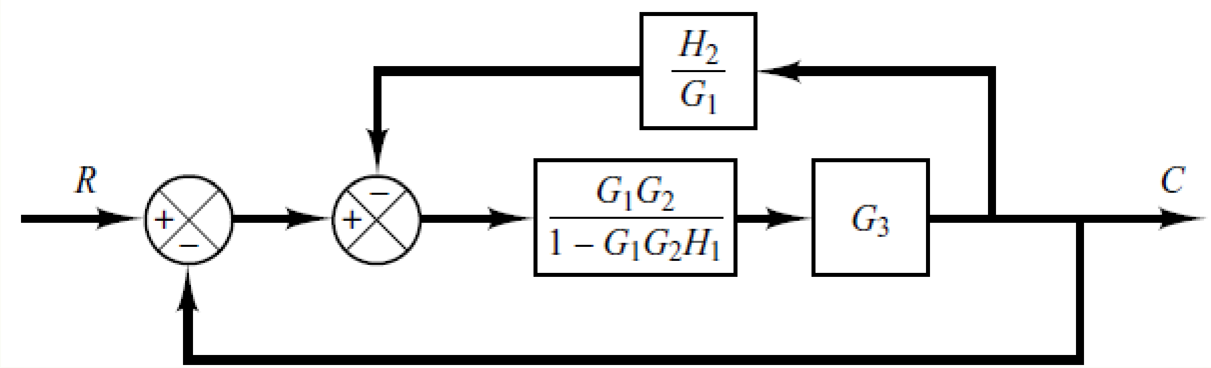


By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1, we obtain Figure

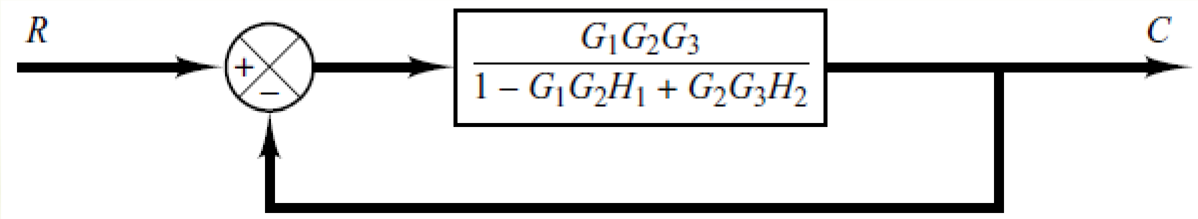


Example-6: Continue

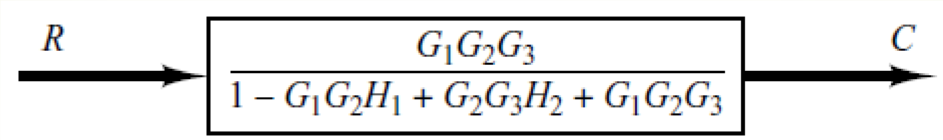
Eliminating the positive feedback loop, we have



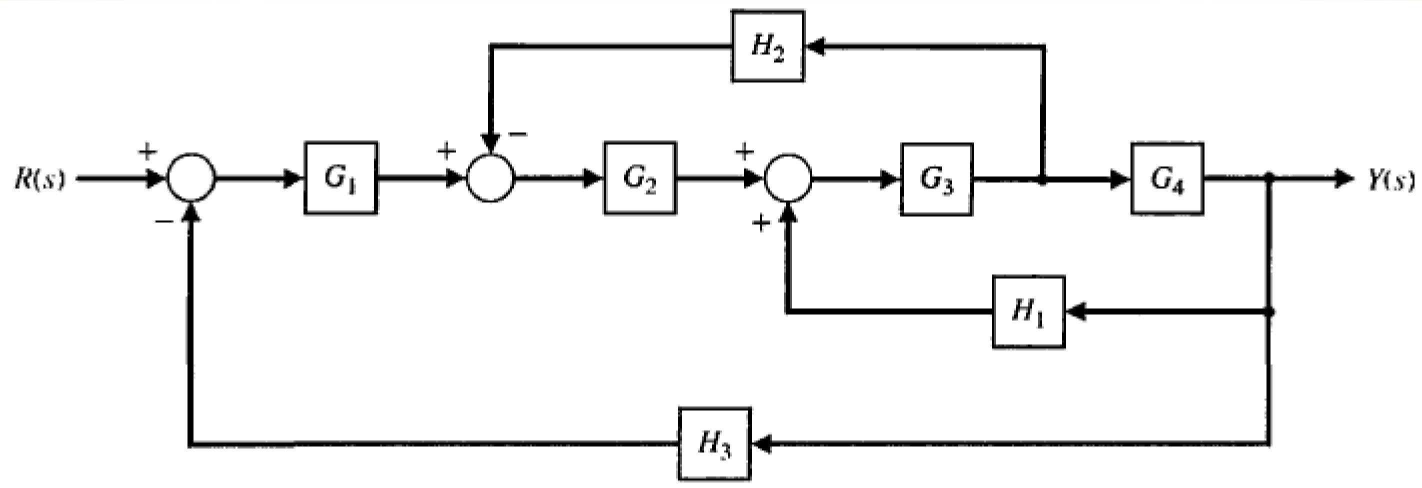
The elimination of the loop containing H_2/G_1 gives



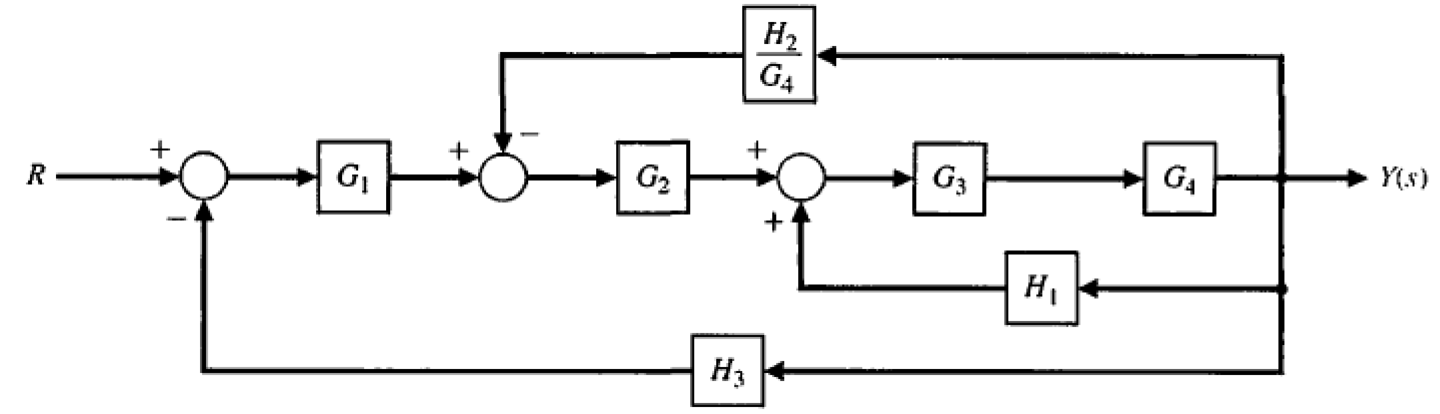
Finally, eliminating the feedback loop results in



Example-7: Reduce the Block Diagram.

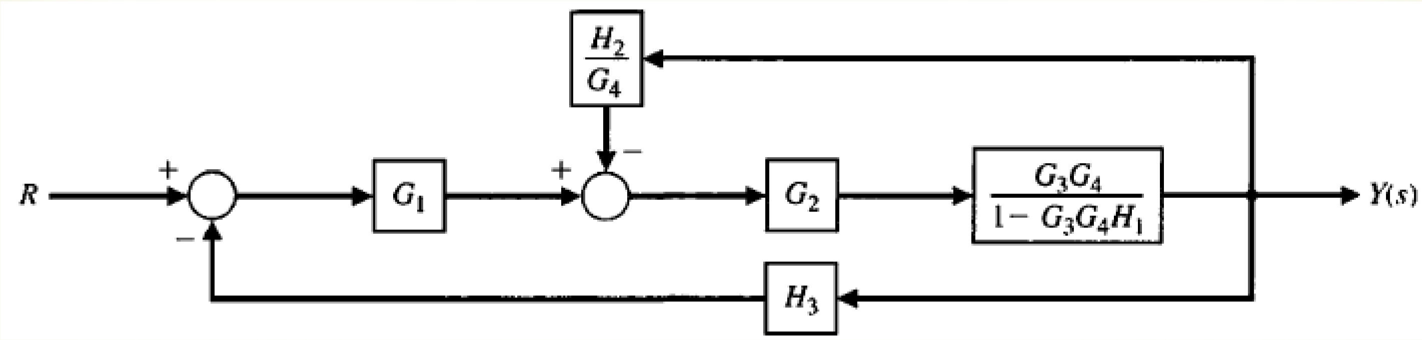


First, to eliminate the loop $G_3G_4H_1$, we move H_2 behind block G_4

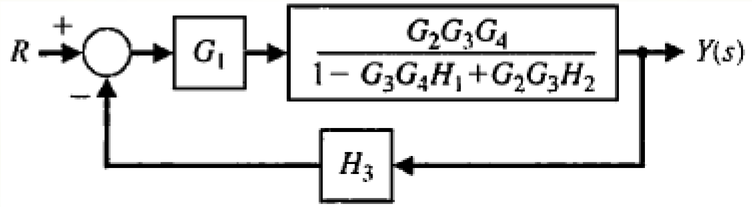


Eliminating the loop $G_3G_4H_1$ we obtain

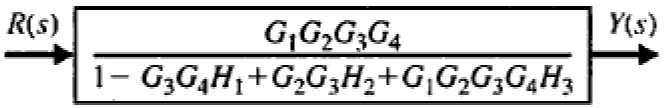
Example-7: Continue



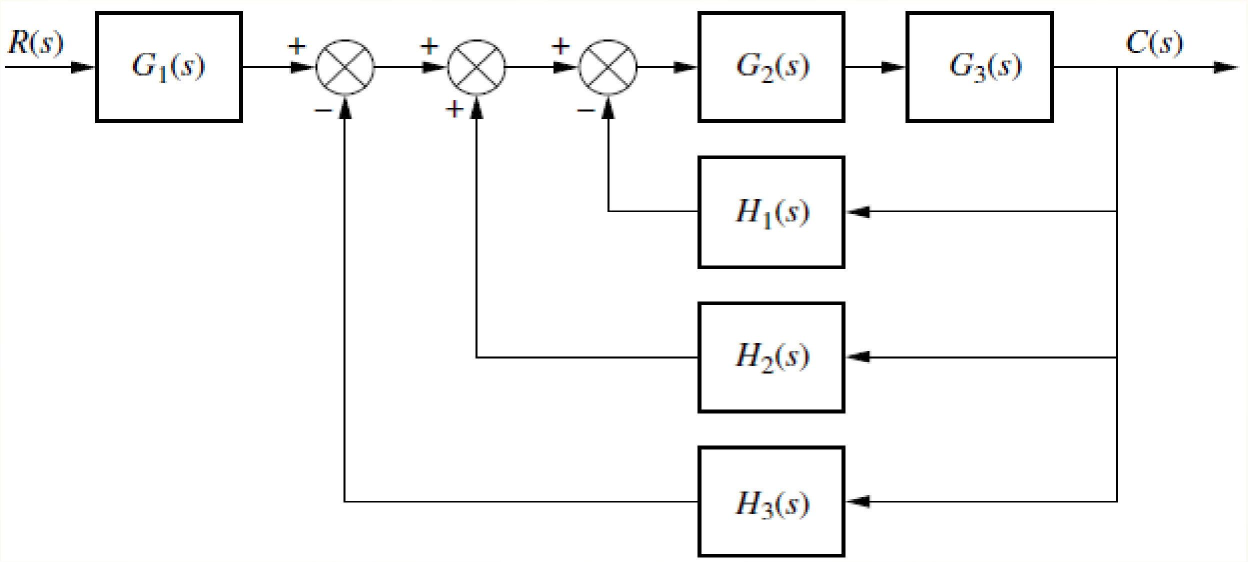
Then, eliminating the inner loop containing H_2/G_4, we obtain



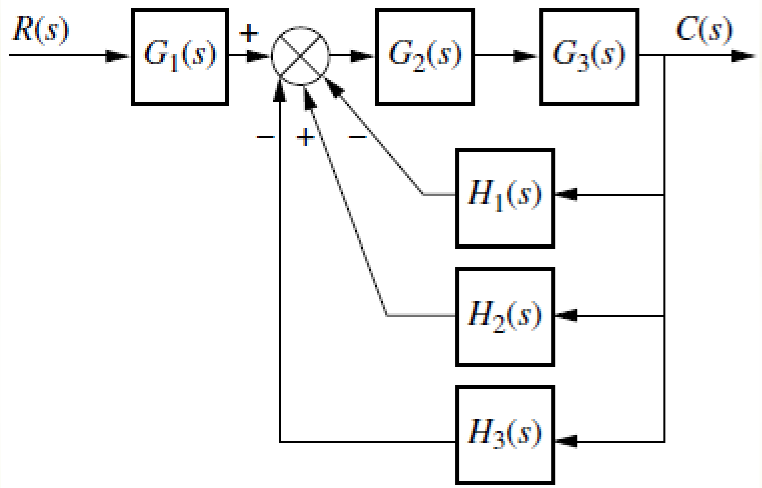
Finally, by reducing the loop containing H_3, we obtain



Example-8: Reduce the Block Diagram (From Nise; page-242).

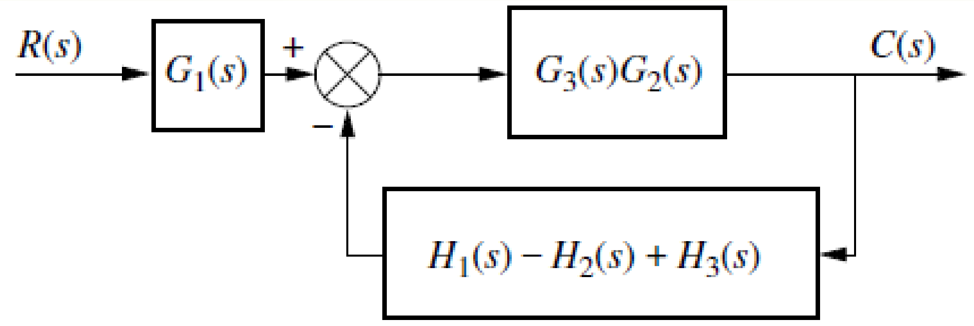


First, the three summing junctions can be collapsed into a single summing junction,

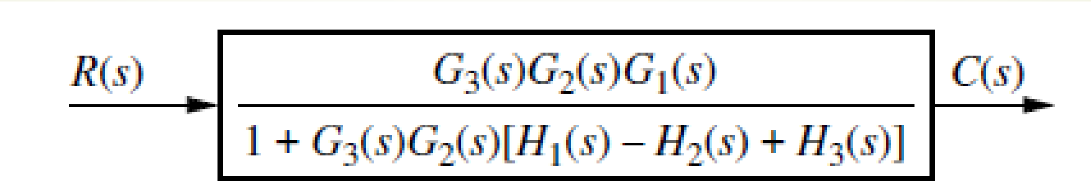


Example-8: Continue

Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade.

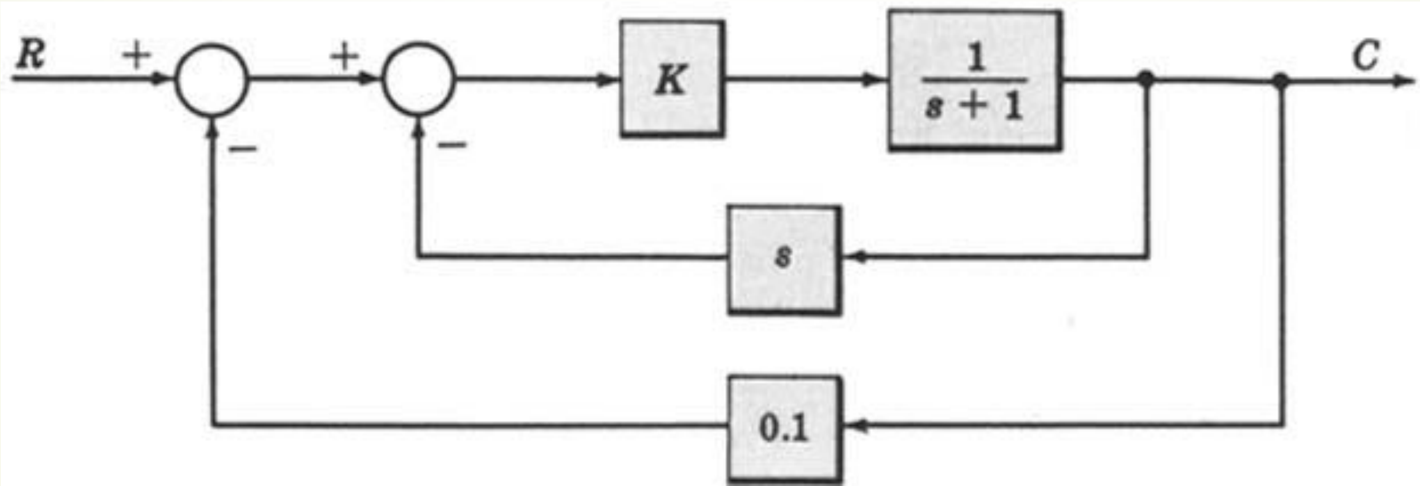


Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure



Example-9: For the system represented by the following block diagram determine:

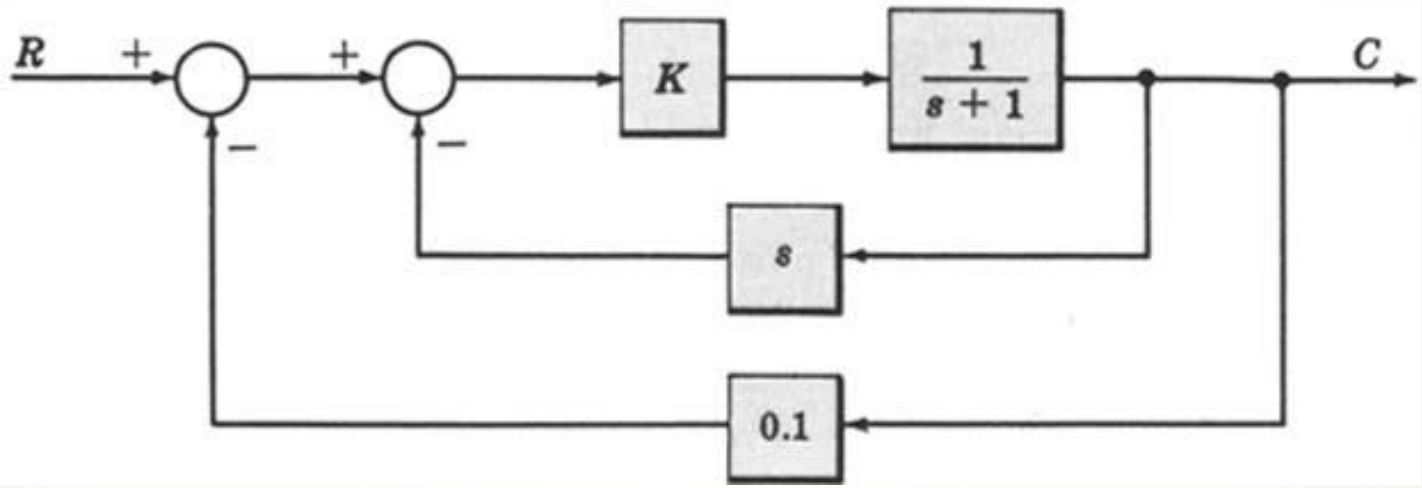
1. Open loop transfer function
2. Feed Forward Transfer function
3. control ratio
4. feedback ratio
5. error ratio
6. closed loop transfer function
7. characteristic equation
8. closed loop poles and zeros if K=10.

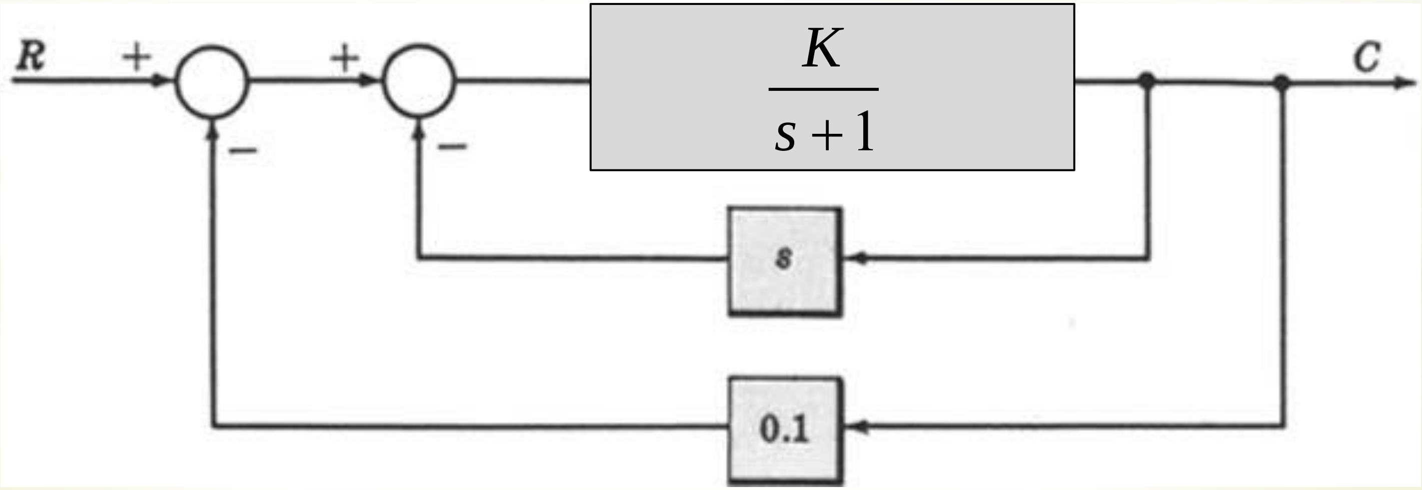


Ebrahim

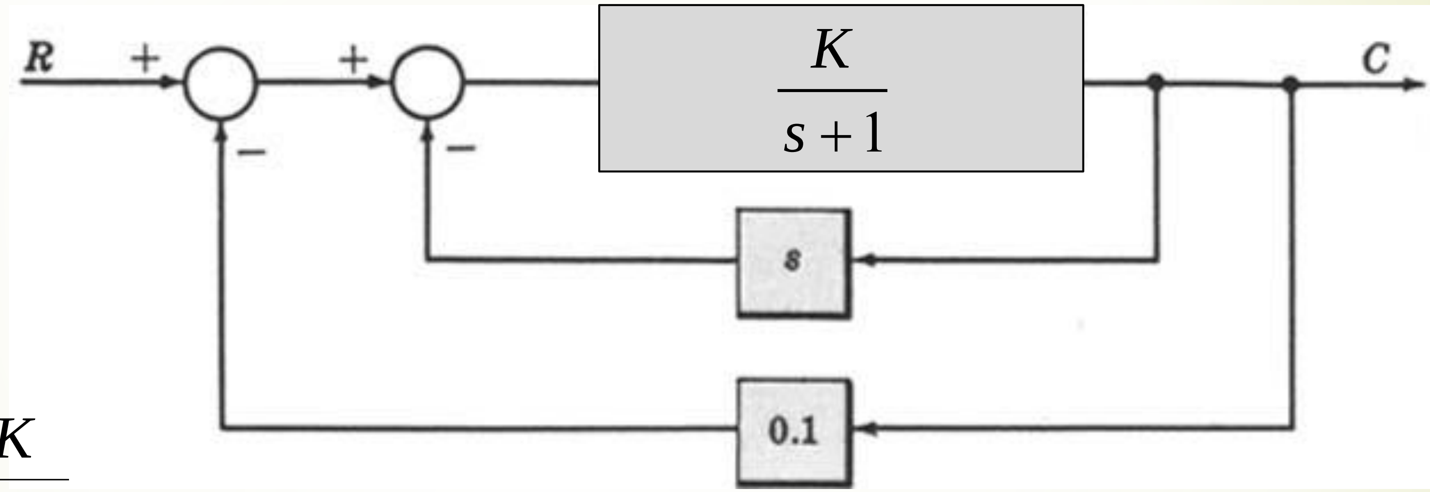
Example-9: Continue

- First we will reduce the given block diagram to canonical form

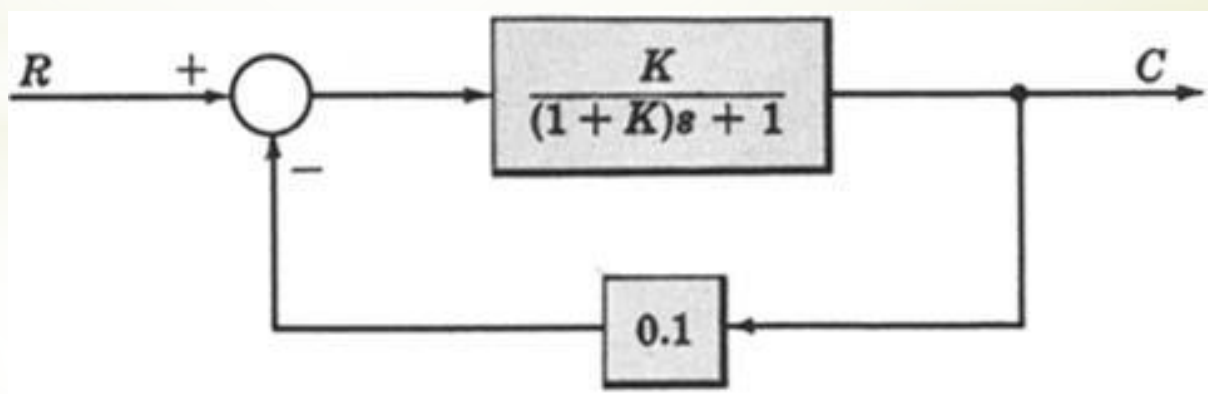




Example-9: Continue



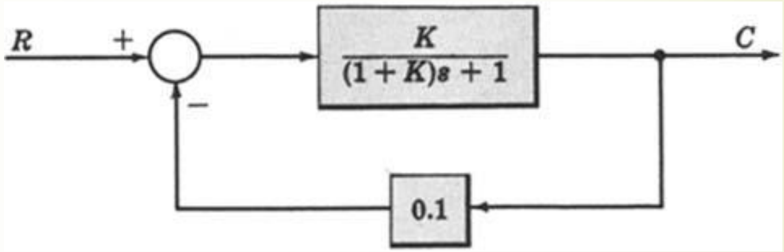
$$\frac{G}{1+GH}=\frac{\dfrac{K}{s+1}}{1+\dfrac{K}{s+1}s}$$



Example-9: Continue

1. Open loop transfer functior $\dfrac{B(s)}{E(s)} = G(s)H(s)$
2. Feed Forward Transfer functi $\dfrac{C(s)}{E(s)} = G(s)$
3. control ratio $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$
4. feedback ratio $\dfrac{B(s)}{R(s)} = \dfrac{G(s)H(s)}{1+G(s)H(s)}$
5. error ratio $\dfrac{E(s)}{R(s)} = \dfrac{1}{1+G(s)H(s)}$
6. closed loop transfer functior $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$
7. characteristic equatio $1+G(s)H(s) = 0$
8. closed loop poles and zeros if K=10

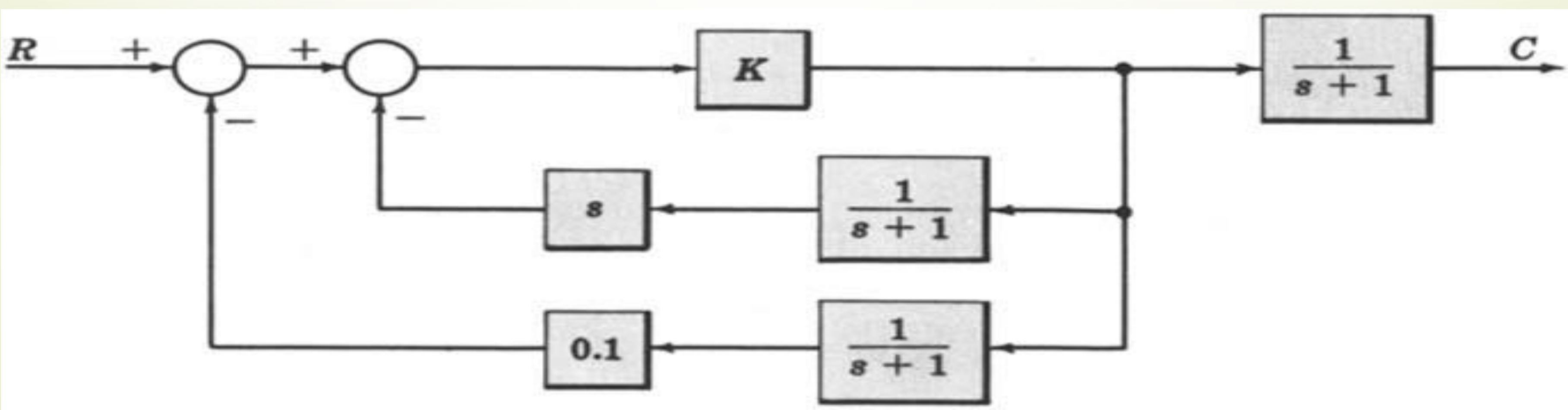
$G(s)$



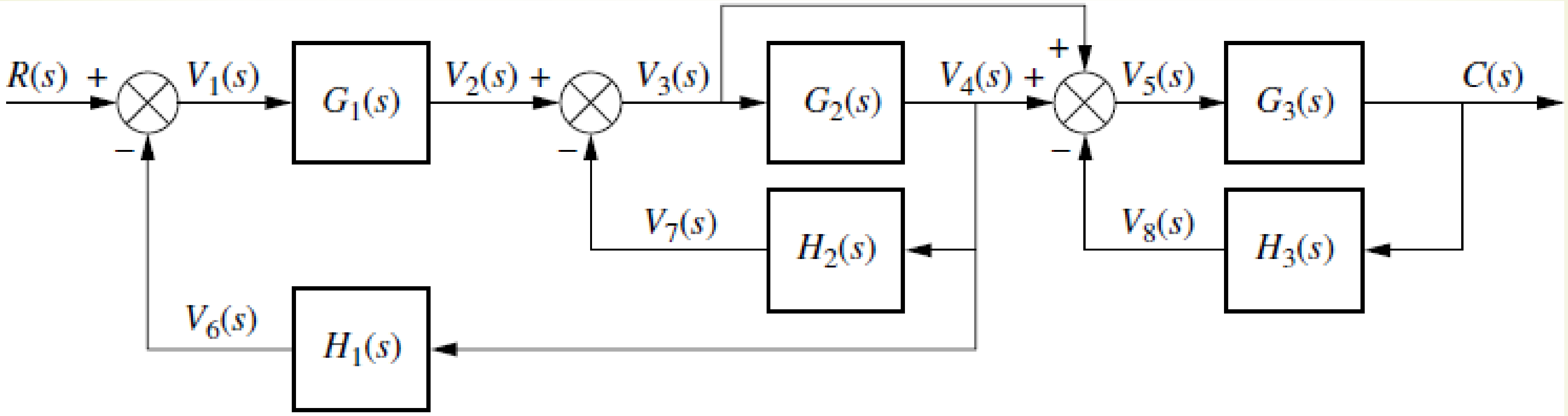
$H(s)$

Example-10: For the system represented by the following block diagram determine:

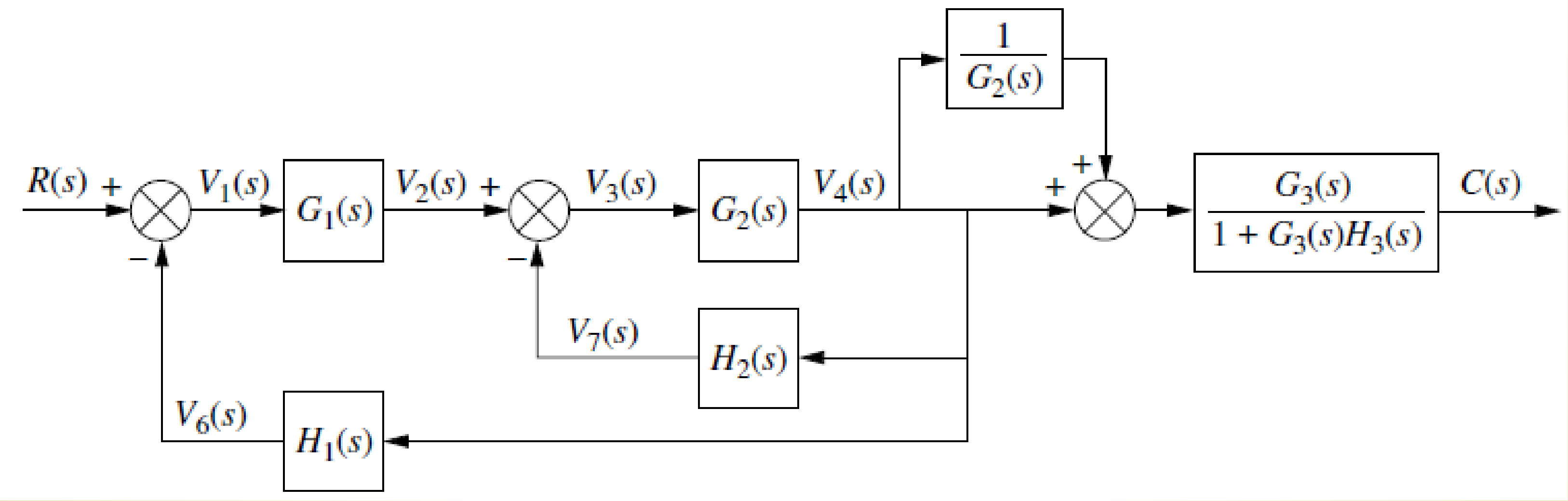
1. Open loop transfer function
2. Feed Forward Transfer function
3. control ratio
4. feedback ratio
5. error ratio
6. closed loop transfer function
7. characteristic equation
8. closed loop poles and zeros if K=100.



* Example-11: Reduce the system to a single transfer function. (from Nise:page-243).

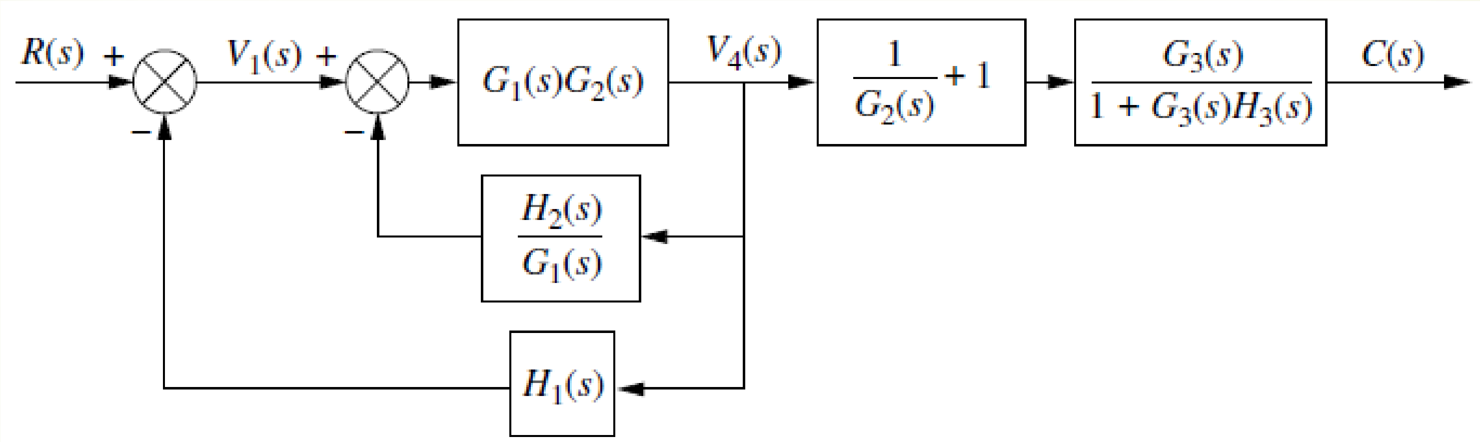


First, move $G_2(s)$ to the left past the pickoff point to create parallel subsystems, and reduce the feedback system consisting of $G_3(s)$ and $H_3(s)$.

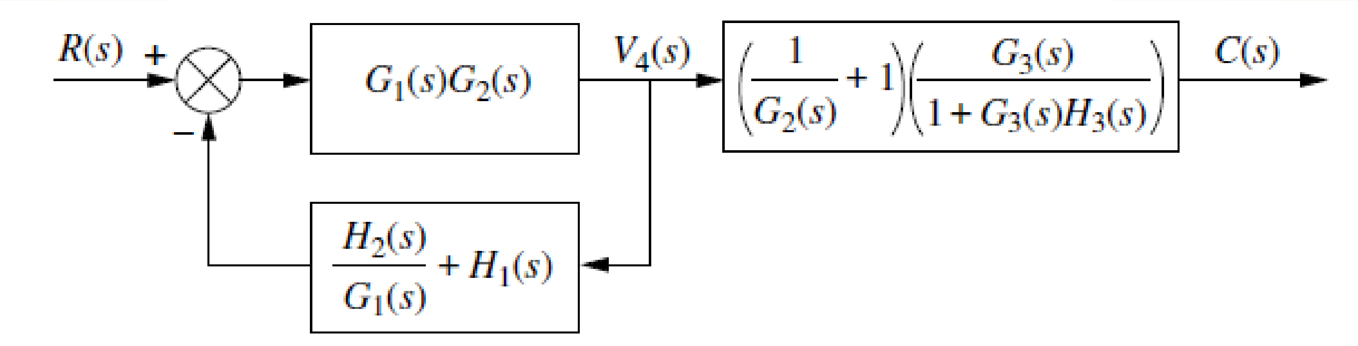


Example-11: Continue

Second, reduce the parallel pair consisting of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past the summing junction, creating parallel subsystems in the feedback.

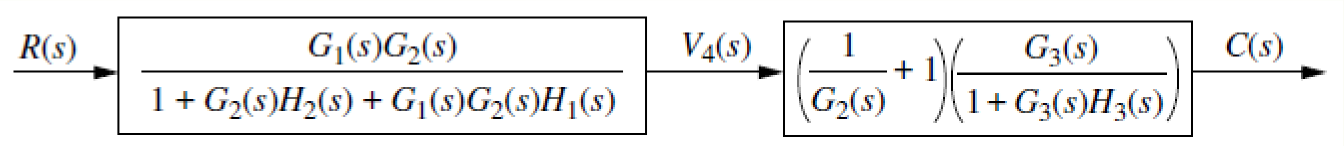


Third, collapse the summing junctions, add the two feedback elements together, and combine the last two cascaded blocks.

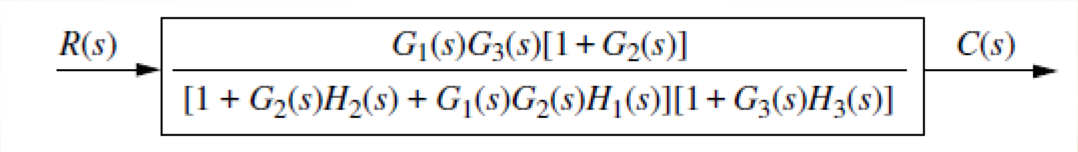


Example-11: Continue

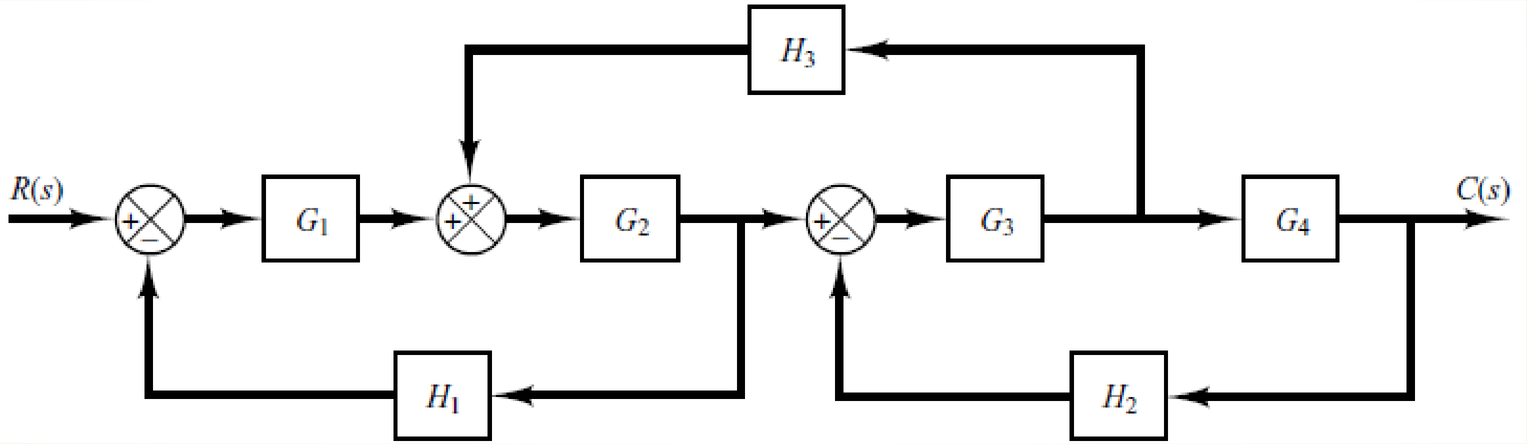
Fourth, use the feedback formula to obtain Figure



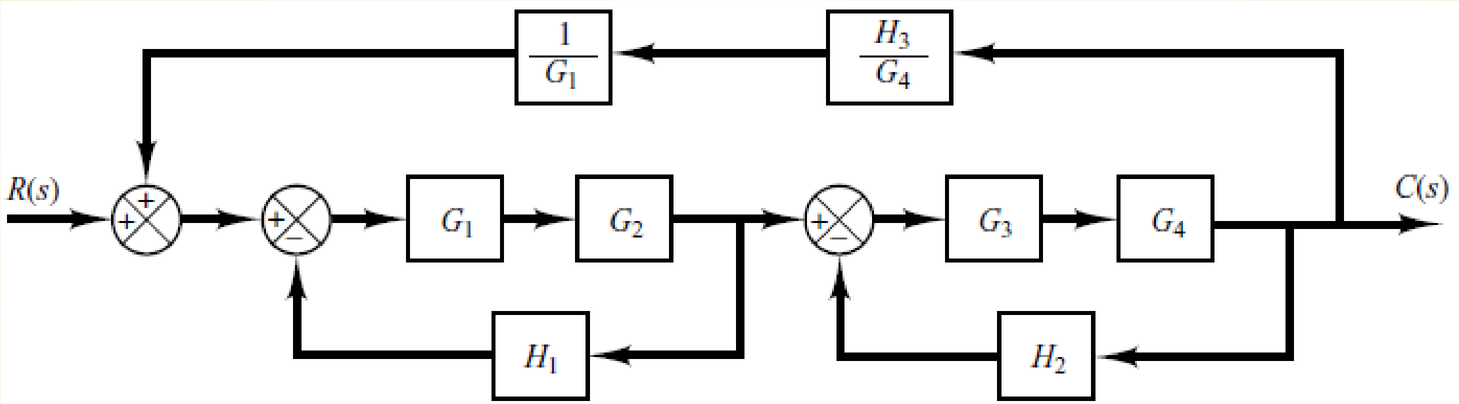
Finally, multiply the two cascaded blocks and obtain the final result,



Example-12: Simplify the block diagram then obtain the close-loop transfer function C(S)/R(S). (from Ogata: Page-47)

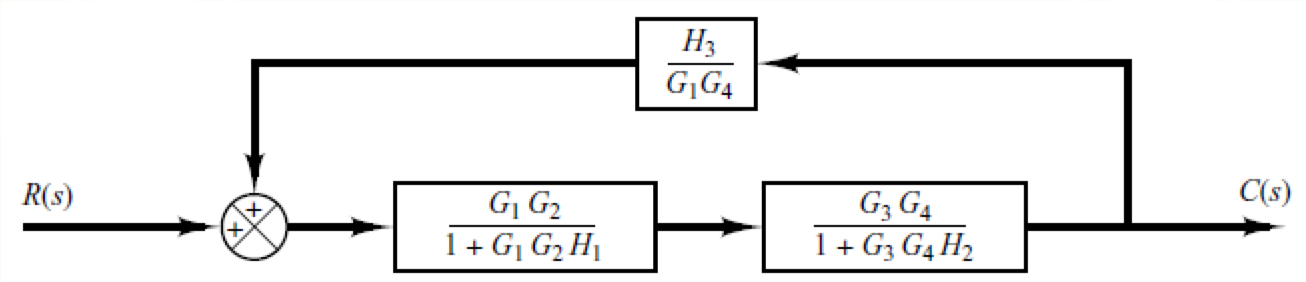


First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.

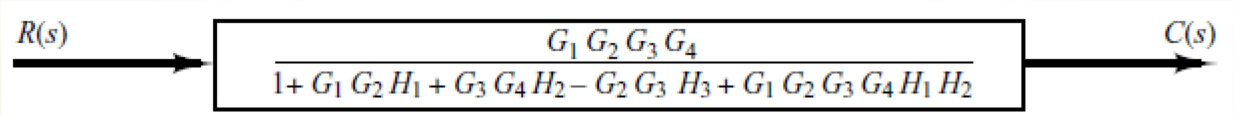


Example-12: Continue

By simplifying each loop, the block diagram can be modified as



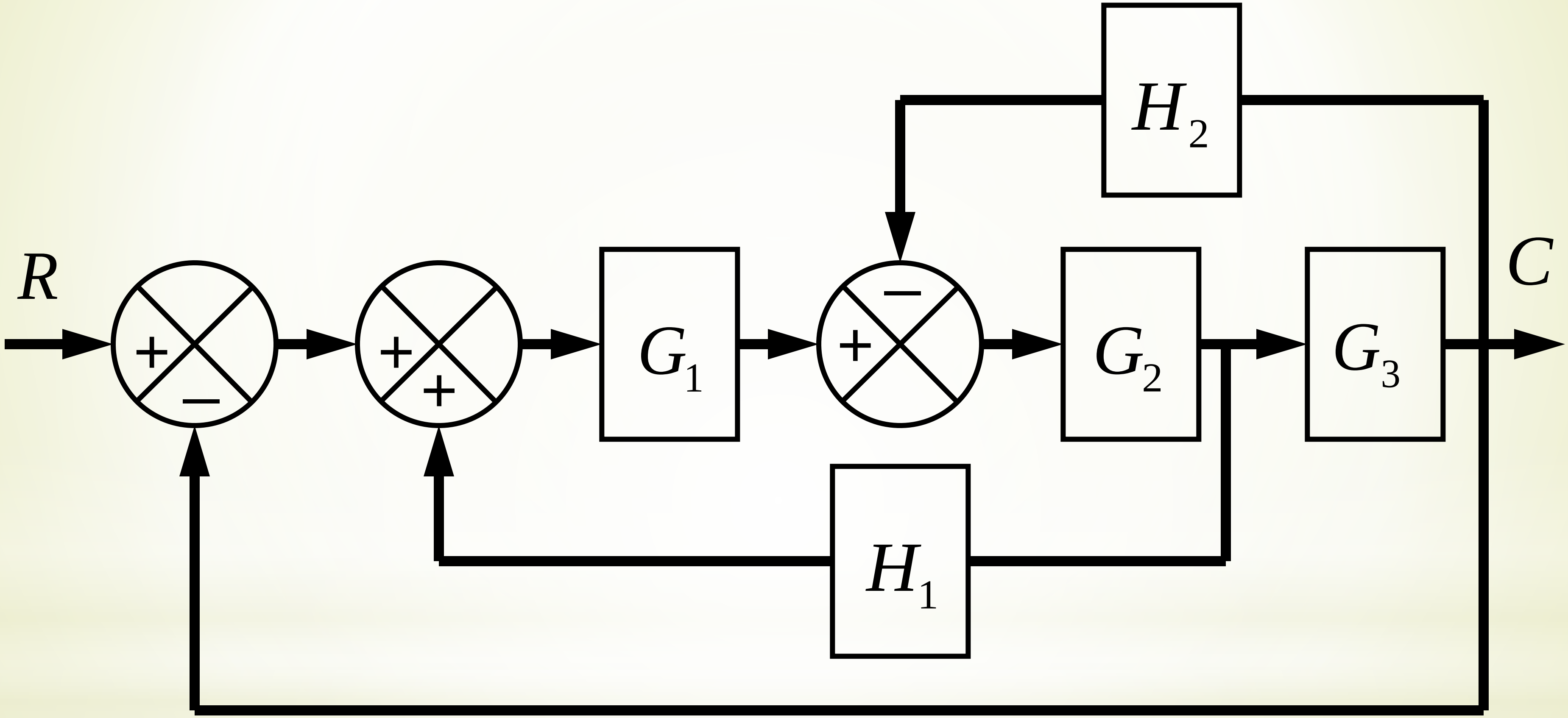
Further simplification results in



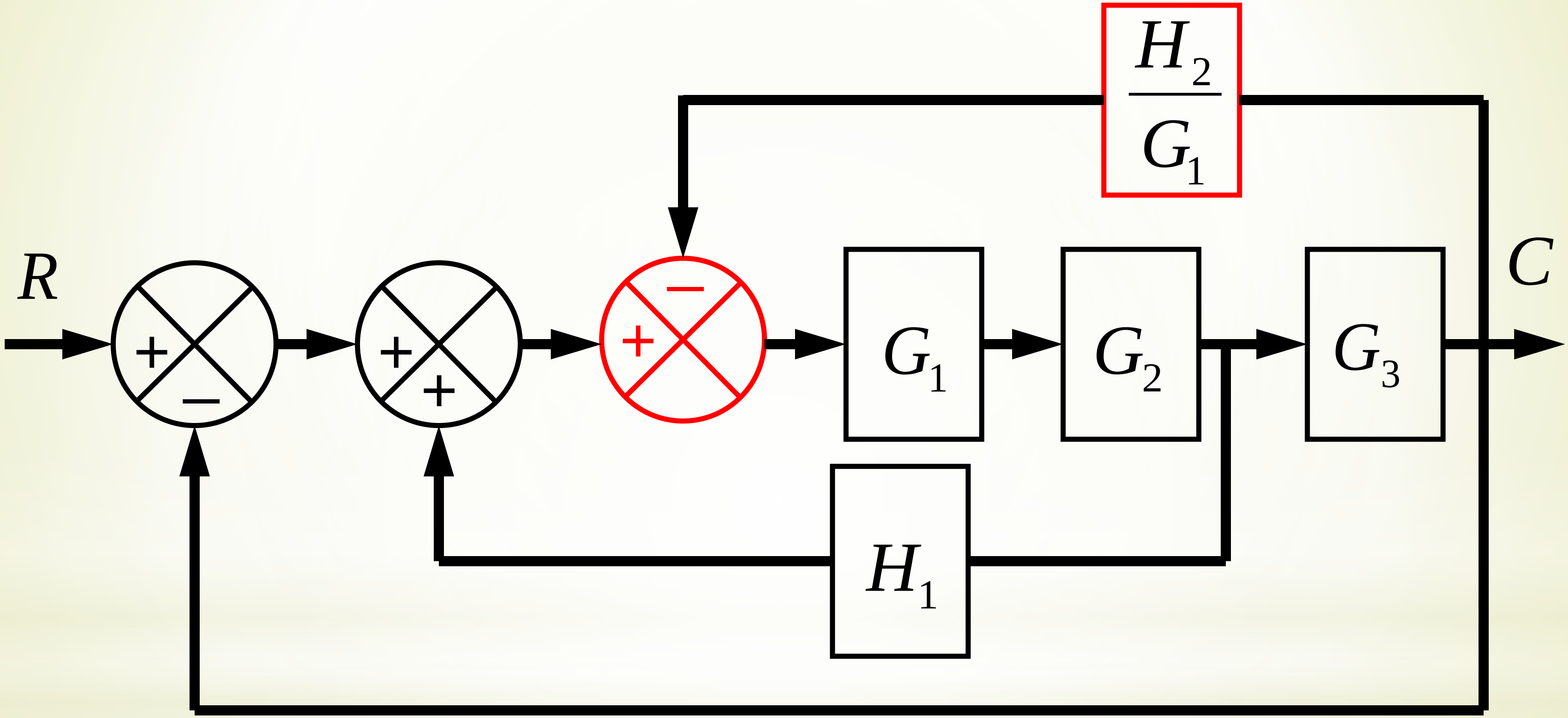
the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$$

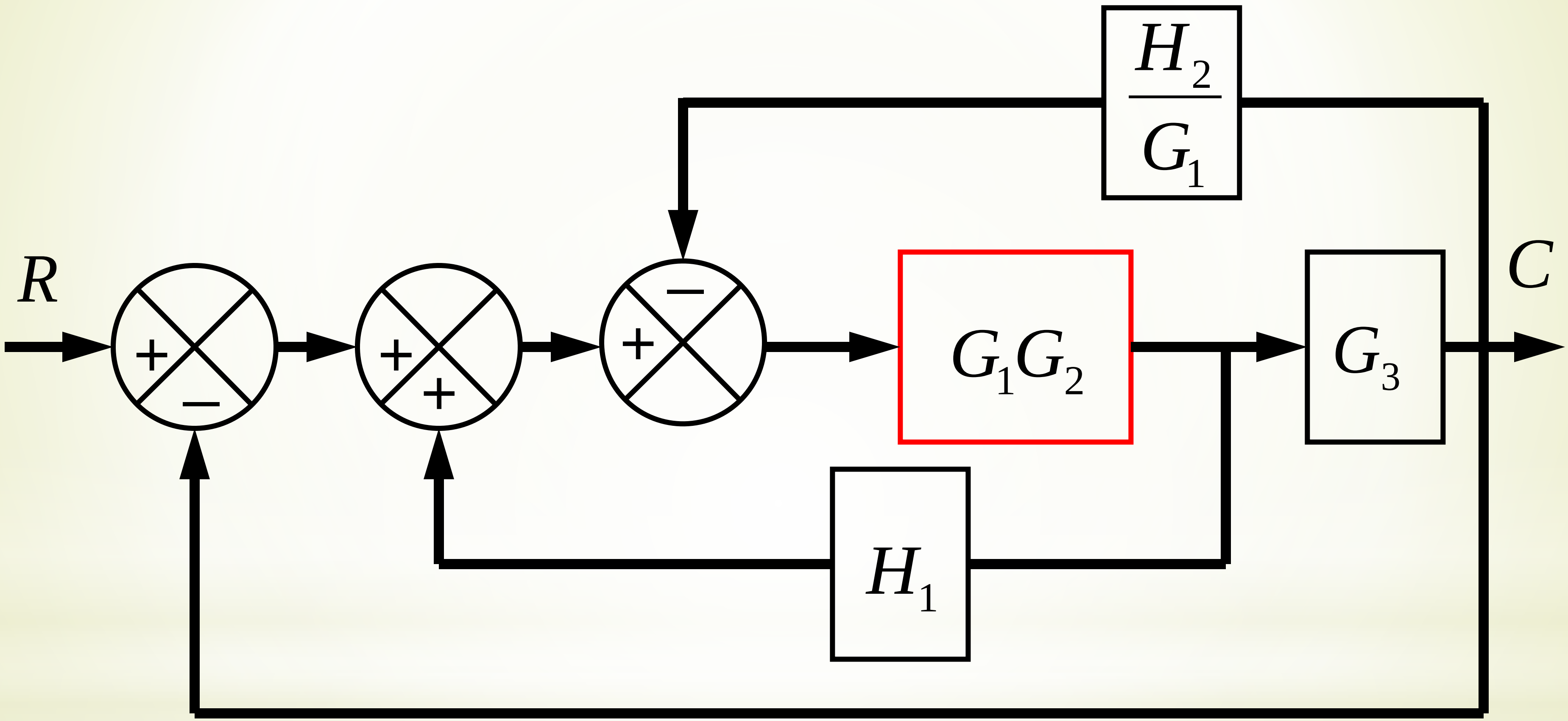
Example-13: Reduce the Block Diagram.



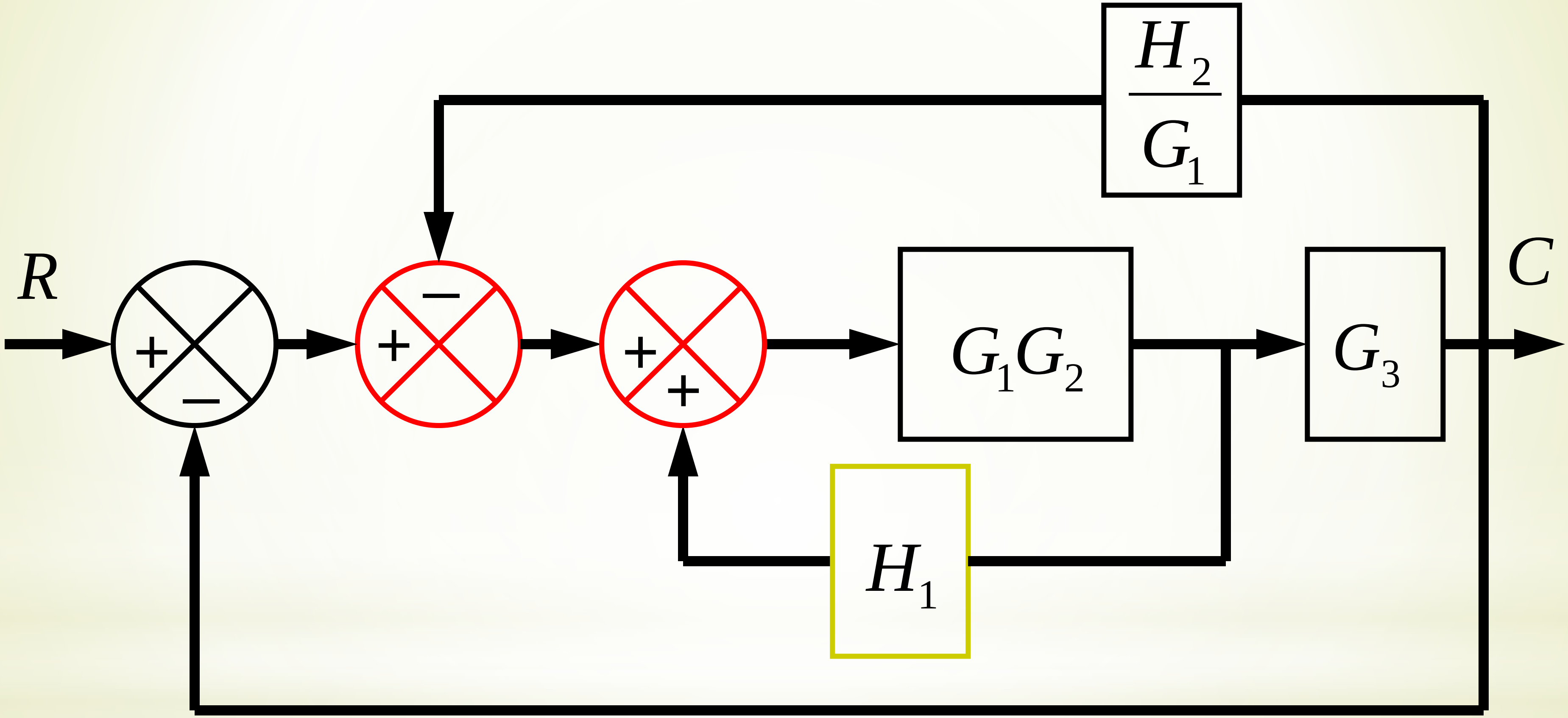
Example-13: Continue



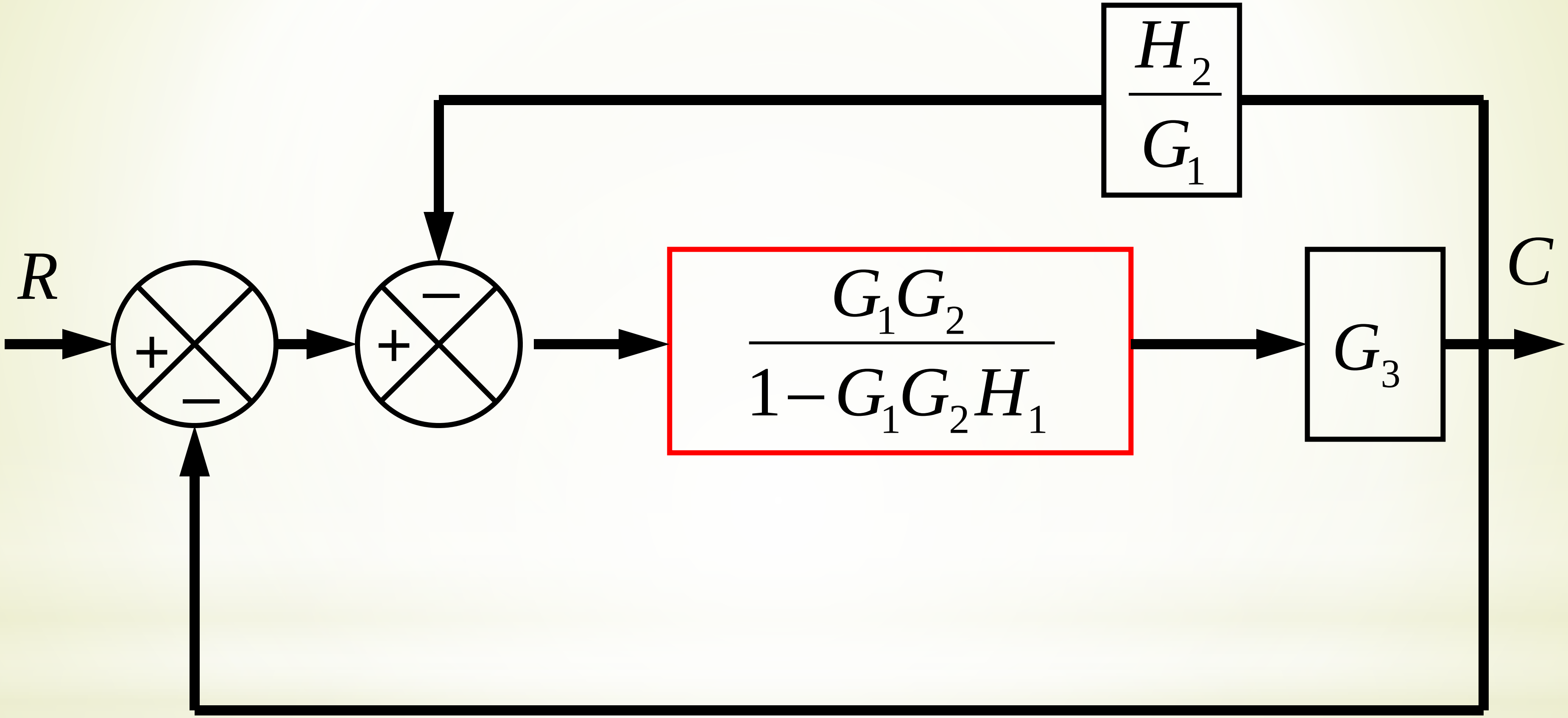
Example-13: Continue



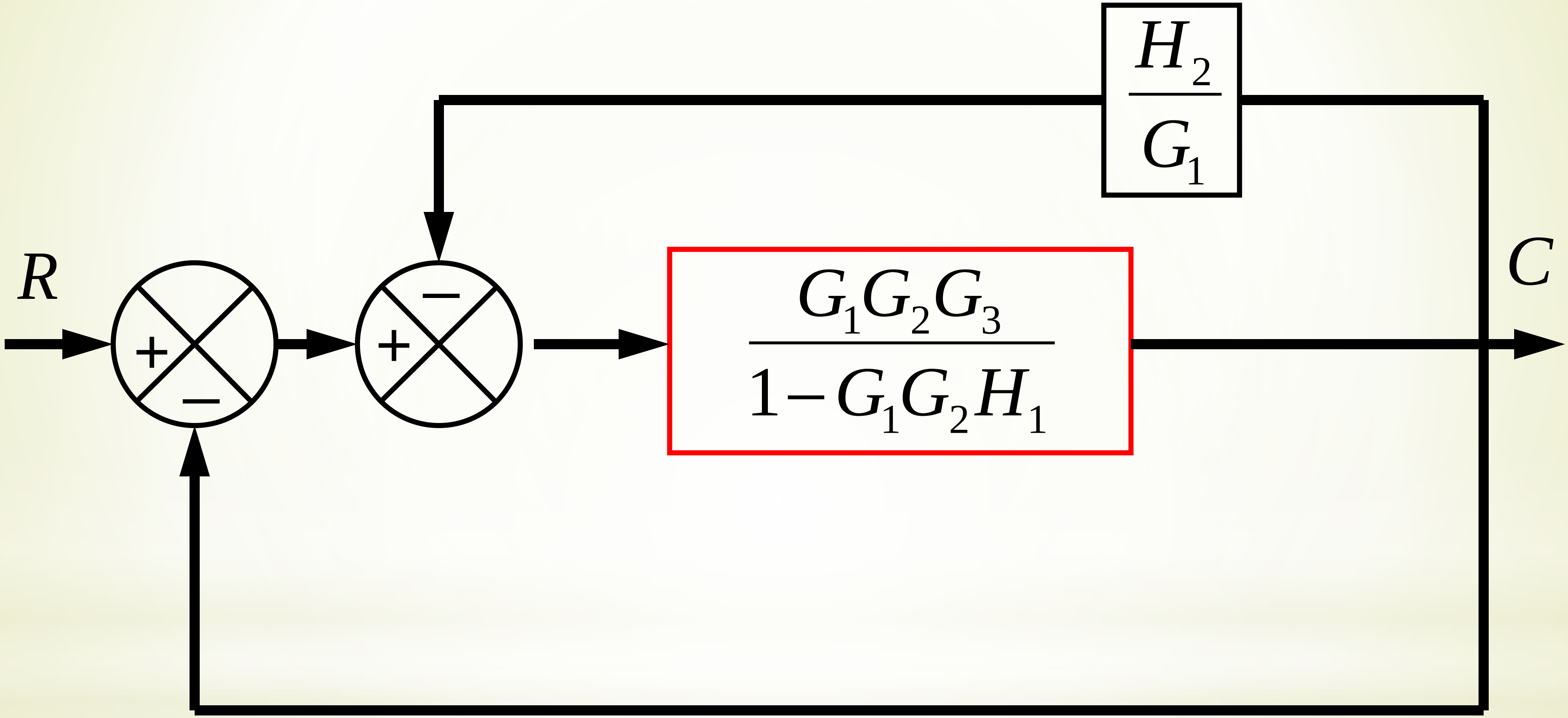
Example-13: Continue



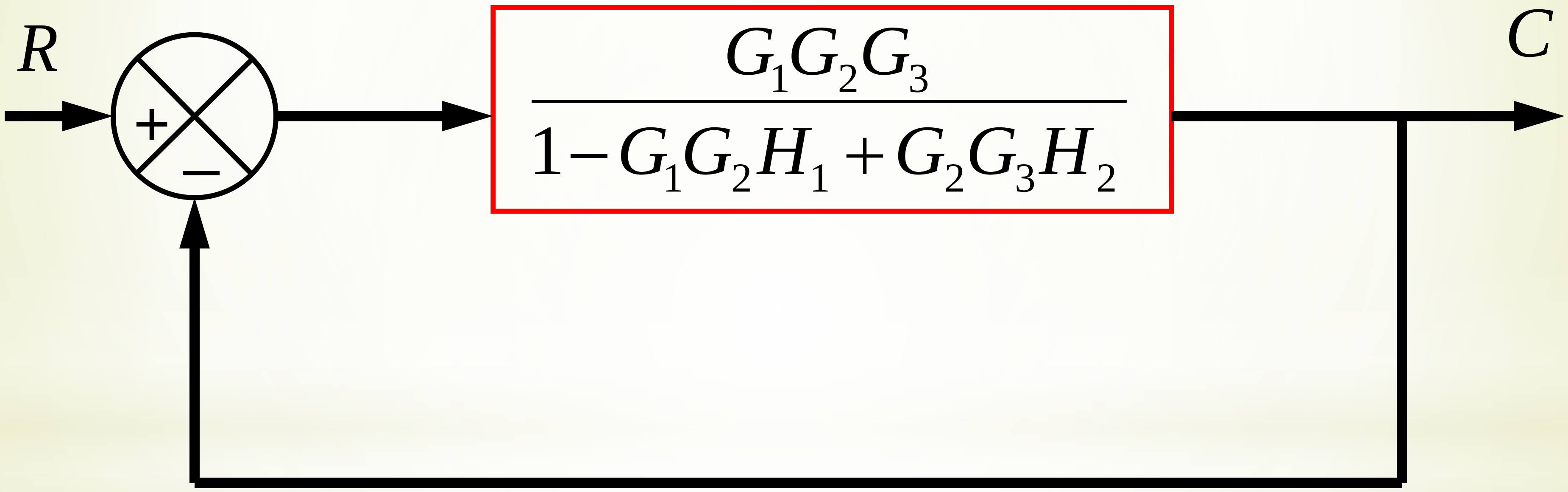
Example-13: Continue



Example-13: Continue



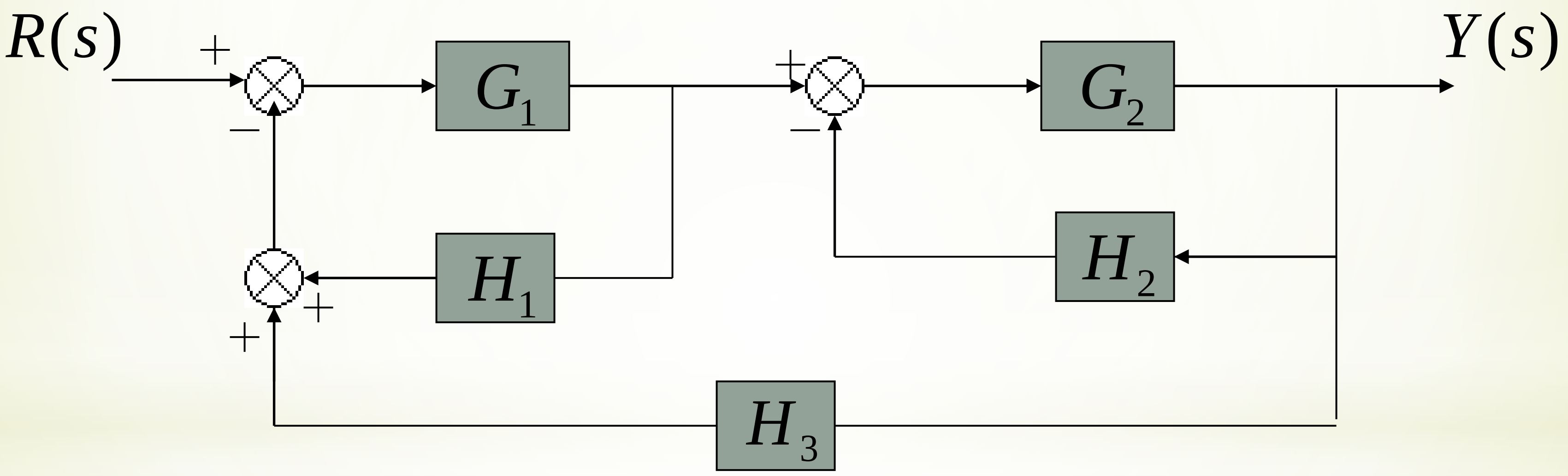
Example-13: Continue



Example-13: Continue

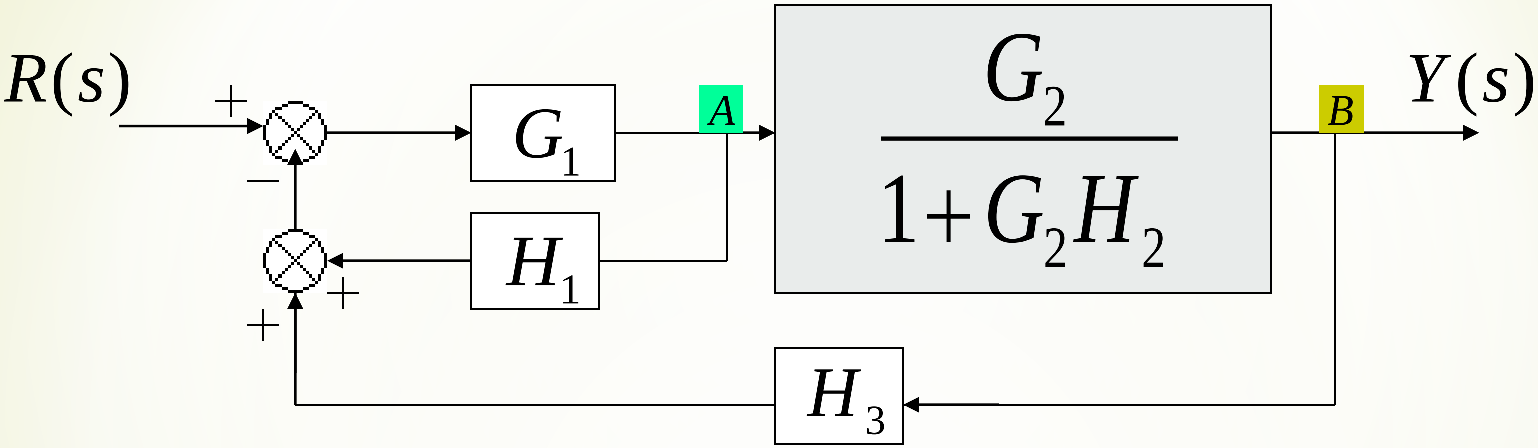
$$R \longrightarrow \boxed{\dfrac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3}} \longrightarrow C$$

Example 14: Find the transfer function of the following block diagrams.

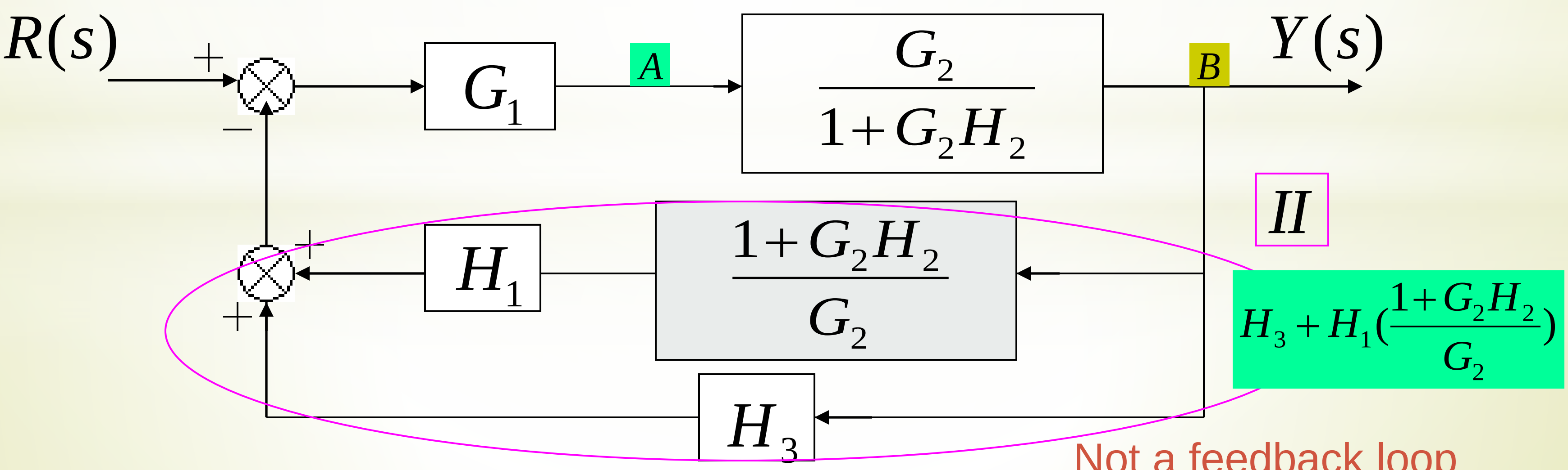


Solution:

1. Eliminate loop I

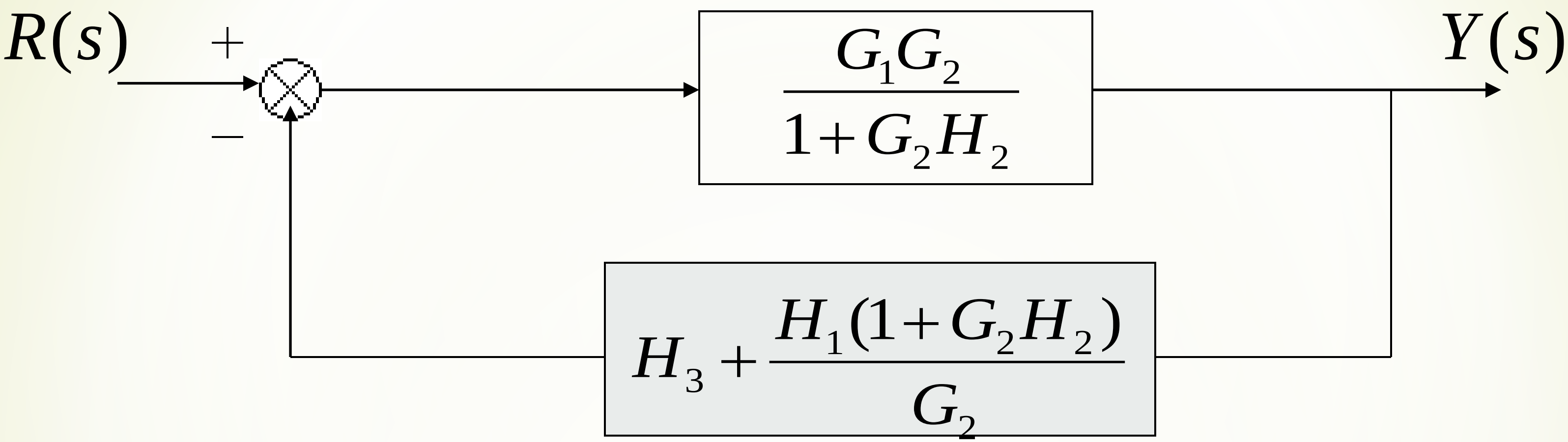


2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



Not a feedback loop

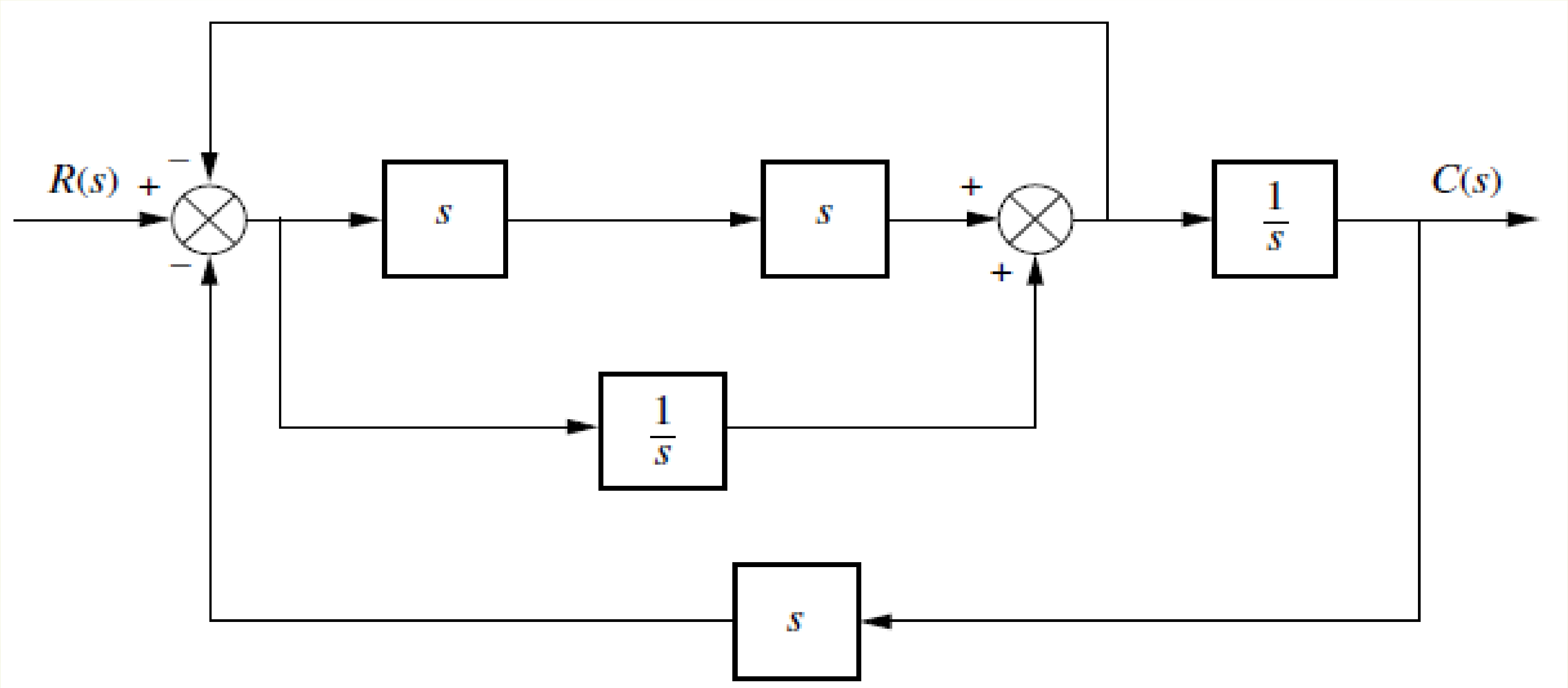
3. Eliminate loop II



$$\frac{Y(s)}{R(s)}=\frac{G_1G_2}{1+G_2H_2+G_1G_2H_3+G_1H_1+G_1G_2H_1H_2}$$

Skill Assessment Exercise:

PROBLEM: Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system



Answer of Skill Assessment Exercise:

ANSWER: $T(s) = \dfrac{s^3 + 1}{2s^4 + s^2 + 2s}$

With Our Best Wishes

Automatic Control (1)

Course Staff

Thank You
For Your Attention

Mohamed Ahmed Ebrahim